3: Transaction Model

Page Model

- All operations on data will be eventually mapped into read and write operations on pages.
- To study the concurrent execution of transactions it is sufficient to inspect the interleavings of the resulting page operations.
- Independently whether a page resides in cache memory or resides on disk, read and write are considered as indivisible.

Parallelism as prerequisite for distributed execution

A transaction T is a partial order $<$[1] of actions in OP, $T = (OP, <)$, where OP is a finite set of T's actions RX and WX, where X is a data item.

Moreover, $< \subseteq OP \times OP$ is a partial order on OP which fulfills the following properties:

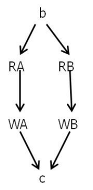
- Each data item is read and written by T at most once.
- If p is a read action and q is a write actions of T and both access the same data item, then $p < q$.

Complete transaction

We call a transaction *complete*, if its first action is begin b and its last action either is commit c or abort a.

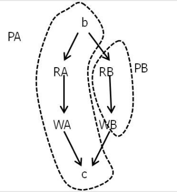
[1] A binary relation is a partial order , if it is reflexive, antisymmetric and transitive.

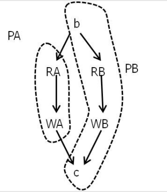
A parallel debit/credit transaction. *b*: BEGIN; *c*: COMMIT.



When transactions are depicted as directed graphs, we omit transitive edges.

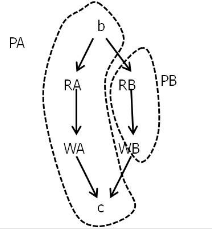
Two parallel debit/credit transactions, each prepared for parallel execution.



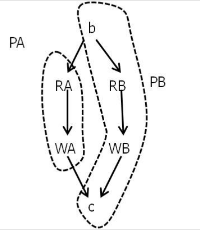


$\Longrightarrow$ Definition of a schedule? Definition of serializability?

Two parallel debit/credit transactions, each prepared for parallel execution.



Transaction T_1



Transaction T_2

Locally observable schedules of the two transactions when executed in parallel by CPU PA and CPU PB.

(i) $PA: \quad R_1A\ W_1A\ R_2A\ W_2A$
$PB: \quad R_1B\ W_1B\ R_2B\ W_2B$

(ii) $PA: \quad R_1A\ W_1A\ R_2A\ W_2A$
$PB: \quad R_2B\ W_2B\ R_1B\ W_1B$

On each CPU in both cases the local schedules are serializable - however, globally, in the second case the transactions are not executed in a serializable manner!

Histories and schedules

Let $\mathcal{T} = \{T_1, \ldots, T_n\}$ be a (finite) set of complete transactions, where for each T_i we have $T_i = (OP_i, <_i)$.

A *history* of $\mathcal{T}$ is a pair $S = (OP_S, <_S)$, where

- $OP_S = \cup_{i=1}^{n} OP_i$ and $<_S$ a partial order on OP_S such that $<_S \supseteq \cup_{i=1}^{n} <_i$.
- Let $p, q \in OP_S$, where p and q belong to distinct transactions, however access the same data object. If p or q is a write action, then either $p <_S q$ or $q <_S p$; we say, p and q are in *conflict*; if $p <_S q$ and p and q are in conflict, we write $(p, q) \in conf(S)$.

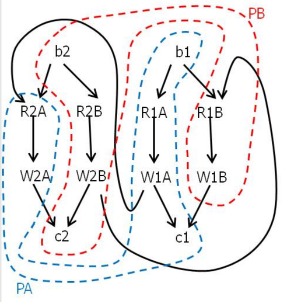
A *schedule* of $\mathcal{T}$ is a prefix of a history.[2]

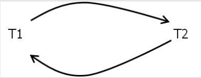
Conflict graph

The conflict graph of a schedule S is given as $G(S) = (V, E)$, where V is the set of transactions in S and the set of edges E is given by the conflicts in S: $T_i \rightarrow T_j \in E$, iff there are conflicting actions $p \in OP_i$, $q \in OP_j$ and $p <_S q$.

[2] A partial order $L' = (A', <')$ is a prefix of a partial order $L = (A, <)$, if $A' \subseteq A$, $<' \subseteq <$, for all $a, b \in A'$: $a <' b$ if $a < b$, and for all $p \in A, q \in A'$: $p < q \Rightarrow p <' q$.

A schedule/history of the two parallel debit/credit transactions.





The schedule is not serializable as its conflict graph is cyclic.

Serializability

- A schedule $S = (OP_S, <_S)$ is *serial*, if for any two transactions T_1, T_2 appearing in S, $<_S$ orders all actions of T_1 before all actions of T_2, or vice versa.
- A schedule is called (conflict-)serializable,[3] if there exists a (conflict-)equivalent serial schedule over the same set of transactions.
- A schedule $S = (OP_S, <_S)$ is serializable, iff its conflict graph is acyclic.

[3] We consider only conflict-serializability and therefore talk about serializability in the sequel, for short.