Course Evaluation

Evaluation period from 01.07. - 19.07.

The course evaluation is now online available at:

https://ilias.uni-freiburg.de/goto.php?target=svy_74197&client_id=unifreiburg
Recap Petri Net basics

\[ N = (P, I, F, V, m_0) \]

\[ T = \{ t_1, t_2 \} \]

\[ m_0 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \]

\[ \Delta t_1 = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \]

\[ \Delta t_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \]

\[ t_1^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

\[ t_2^+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]
Chapter 7: Modeling and Analysis of Distributed Applications

Petri-Nets

- Petri-nets are abstract formal models capturing the flow of information and objects in a way which makes it possible to describe distributed systems and processes at different levels of abstraction in a unified language.
- Petri-nets have the name from their inventor Carl Adam Petri, who introduced this formalism in his PhD-thesis 1962.
Processing of complaints: informal description.

- Customer inquiring
- Archive
- Complaint registration
- Complaint processing
Complaints processing: formal Petri-net orchestration.\textsuperscript{1}

Complaints processing: more than one complaint
Complaints processing: how to distinguish complaints
Complaints processing: keeping things together

[Diagram of Petri-net]
Petri-nets

Petri-nets model system dynamics.

- Activities trigger state transitions,
- activities impose control structures,
- applicable for modelling discrete systems.

Benefits

- Uniform language,
- can be used to model sequential, causal independent (concurrent, parallel, nondeterministic) and monitored exclusive activities.
- open for formal analysis, verification and simulation,
- graphical intuitive representation.

The name *Petri-net* denotes a variety of different versions of nets - we will discuss the special case of *System Nets* following the naming introduced by W. Reisig.
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Section 7.1 Elementary System Nets

Basic elements of an elementary System Net (eS-Net)

- System states are represented by *places*, graphically circles or ovals.
- A place may be marked by an arbitrary number of *tokens* graphically represented by black dots.
- System dynamics is represented by *transitions*, graphically rectangles.
- *Transitions* represent activities (events) and the causalities between such activities (events) are represented by edges.
- *Multiplicities* represent the consumption, respectively creation of resources which are caused by the *occurrence* of activities.
Section 7.1 Elementary System Nets

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3-Philosopher-Problem

\( b_j \): philosopher starts eating; \( e_j \): philosopher stops eating; 
\( i_j \): philosopher is eating; \( g_j \): fork on the desk; 
\( 1 \leq j \leq 3 \).
A transition may occur when certain conditions with respect to the markings of its directly connected places are fulfilled; the occurrence of a transition - also called its firing - effects the markings of its directly connected edges, i.e. has local effects.

The surrounding of a transition \( t \) is given by \( t \) and all its directly connected places:

\[ s_1, \ldots, s_k \text{ are called preconditions (pre-places), } s_{k+1}, \ldots, s_n \text{ postconditions (post-places).} \]

A place which is pre- and post-place at the same time is called a loop.
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The surrounding of a transition $t$ is given by $t$ and all its directly connected places:

\[
\begin{array}{c}
\text{s}_1 \\
\vdots \\
\text{s}_k \\
\leftrightarrow \\
\text{s}_{k+1} \\
\vdots \\
\text{s}_n
\end{array}
\]

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A place which is pre- and post-place at the same time is called a *loop*. 
A *net* is given as a tripel \( N = (P, T, F) \), where

- \( P \), the set of *places*, and \( T \), the set of *transitionen*, are non-empty disjoint sets,
- \( F \subseteq (P \times T) \cup (T \times P) \), is the set of directed edges, called *flow relation*, which is a binary relation such that \( \text{dom}(F) \cup \text{cod}(F) = P \cup T \).

Let \( N = (P, T, F) \) be a net and \( x \in P \cup T \).

\[
x F := \{ y \mid (x, y) \in F \}
F x := \{ y \mid (y, x) \in F \}
\]

For \( p \in P \), \( p F \) is the set of *post-transitions* of \( p \); \( F p \) is the set of *pre-transitions* of \( p \).
For \( t \in T \), \( t F \) is the set of *post-places* of \( t \); \( F t \) is the set of *pre-places* of \( t \).
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\begin{align*}
xE & := \{ y \mid (x, y) \in F \} \\
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\end{align*}
\]

For \( p \in P \), \( pF \) is the set of post-transitions of \( p \); \( Fp \) is the set of pre-transitions of \( p \).
For \( t \in T \), \( tF \) is the set of post-places of \( t \); \( Ft \) is the set of pre-places of \( t \).
Let $N = (P, T, F)$ be a net. Any mapping $m$ from $P$ into the set of natural numbers $\text{NAT}$ is called a *marking* of $P$.

A mapping $P \rightarrow \text{NAT} \cup \{\omega\}$ is called $\omega$-*marking*. $\omega$ represents an infinitely large number of tokens.

Arithmetic of $\omega$:

$$\omega - n = \omega, \omega + n = \omega, n \cdot \omega = \omega, 0 \cdot \omega = 0, \omega > n$$

where $n \in \text{NAT}, n > 0$.

A *marking* represents a possible system state.
Let $N = (P, T, F)$ be a net. Any mapping $m$ from $P$ into the set of natural numbers $\text{NAT}$ is called a marking of $P$.

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A eS-Net is given as \( N = (P, T, F, V, m_0) \), where
- \((P, T, F)\) a net,
- \( V : F \rightarrow \text{NAT}^+ \) a multiplicity,
- \( m_0 \) a marking called initial marking.

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A transition may fire once it is enabled.

Let \( N = (P, T, F, V, m_0) \) a eS-Net, \( m \) a marking and \( t \in T \) a transition.

- \( t \) is enabled at \( m \), if for all pre-places \( p \in Ft \) there holds:
  \[
  m(p) \geq V(p, t).
  \]

Whenever \( t \) is enabled at \( m \), then \( t \) may fire at \( m \). Firing \( t \) at \( m \) transforms \( m \) to \( m' \), \( m'[t > m'] \), in the following way:

\[
m'(p) := \begin{cases} 
  m(p) - V(p, t) + V(t, p) & \text{falls } p \in Ft, p \in tF, \\
  m(p) - V(p, t) & \text{falls } p \in Ft, p \not\in tF, \\
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Transitions and markings in terms of vectors

Let places in $P$ be linearly ordered.

- Markings of a net can be considered as vectors of nonnegative integers of dimension $|P|$, called *place-vectors*.
- Transitions $t$ can be characterized as vectors of nonnegative integers of dimension $|P|$, called *transition vectors* $\Delta t, t^+, t^-$:

Let $N = (P, T, F, V, m_0)$ a eS-Net, $p \in P$ and $t \in T$.

$$
t^+(p) := \begin{cases} 
V(t, p) & \text{if } p \in tF, \\
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t^-(p) := \begin{cases} 
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Place and transition vectors at work:

- $m \leq m'$, if $m(p) \leq m'(p)$ for $\forall p \in P$,
- $m < m'$, if $m \leq m'$, however $m \neq m'$.
- $t$ is enabled at $m$ iff $t^- \leq m$,
- $m[t \succ m']$ iff $t^- \leq m$ and $m' = m + \Delta t$. 


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- $m[t > m'$ if $t^- \leq m$ and $m' = m + \Delta t$. 

\[ \Delta t = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]
Reachability

Let \( N = (P, T, F, V, m_0) \) a eS-Net.

We denote \( W(T) \) the set of words with finite length over \( T \); \( \epsilon \in W(T) \) is called the *empty word*.

The length of a word \( w \in W(T) \) is given by \( l(w) \). We have \( l(\epsilon) = 0 \).

Let \( m, m' \) be markings of \( P \) and \( w \in W(T) \). We define a relation \( m[w \succ m'] \) inductively:

- \( m[\epsilon \succ m'] \text{ iff } m = m' \),
- Let \( t \in T, w \in W(T) \). \( m[wt \succ m'] \text{ iff } \exists m'' : m[w \succ m'', m''[t \succ m'] \).

The *reachability relation* \( \succ \) of \( N \) is defined by

\[
m[\star \succ m'] \text{ iff } \exists w : w \in W(T), m[w \succ m'];
\]

\( m' \) is reachabe from \( m \) in \( N \).
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7. Petri-Nets

7.1. Elementary System Nets

$R_N(m) := \{m' \mid m[\ast \succ m']\}$, the set of markings reachable from $m$ by $N$,

$L_N(m) := \{w \mid \exists m' : m[w \succ m']\}$, the set of all words representing firing sequences of transitions of $N$ starting at $m$,

$\Delta w := \sum_{i=1}^{n} \Delta t_i$, where $w = t_1 t_2 \ldots t_n$.

Results

- $[\ast \succ]$ is reflexive and transitive.
- $m[w \succ m'] \Rightarrow (m + m^*)[w \succ (m' + m^*)], \forall m^* \in \text{NAT}^P$. (Monotony)
- $m[w \succ m'] \Rightarrow m' = m + \Delta w$. 
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7. Petri-Nets

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- $R_N(m) := \{ m' | m \presucc m' \}$, the set of markings reachable from $m$ by $N$,
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Results

- $\presucc$ is reflexive and transitive.
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- $m \presucc m' \Rightarrow m' = m + \Delta w$. 

Distributed Systems Part 2 Transactional Distributed Systems Dr.-Ing. Thomas Hornung
Let \( N = (P, T, F, V, m_0) \) a eS-Net. The **Reachability graph** of \( N \) is a directed graph \( \text{EG}(N) := (R_N(m_0), B_N) \); \( R_N(m_0) \) is the set of nodes and \( B_N \) is the set of annotated edges as follows:

\[
B_N = \{ (m, t, m') \mid m, m' \in R_N(m_0), t \in T, m[t \succ m'] \}.
\]
Exercise: Give the reachability graph of the following eS-Net:

\[
R_N(m_0) = \{ (1, 0, 0, 0), (1, 1, 0, 0), (1, 2, 0, 0), (1, 3, 0, 0), \ldots, \\
(0, 0, 1, 0), (0, 1, 1, 0), (0, 2, 1, 0), (0, 3, 1, 0), \ldots, \\
(0, 0, 1, 1), (0, 1, 1, 1), (0, 0, 1, 2), (0, 2, 1, 1), (0, 1, 1, 2), (0, 0, 1, 3), \ldots \}
\]

\[
L_N(m_0) = \{ \epsilon, t_1, t_1 t_1, t_1 t_1 t_1, \ldots, \\
t_2, t_1 t_2, t_1 t_1 t_2, t_1 t_1 t_1 t_2, \ldots, \\
t_1 t_2 t_3, t_1 t_1 t_2 t_3, t_1 t_1 t_1 t_2 t_3, t_1 t_1 t_2 t_1 t_2,$$t_3 t_3, t_1 t_1 t_1 t_2 t_3 t_3 t_3, \ldots \}
\]
Section 7.2 Control Patterns

- eS-nets can be used to model causal dependencies; for modelling temporal aspects extensions of the formalism are required.

- Whenever between some transitions there are no causal dependencies, the transitions are called concurrent; concurrency is a prerequisite for parallelism.
Some typical causalities

Sequence

Iteration
AND-join, OR-join, AND-split, OR-split

AND-join

OR-join

AND-split

OR-split
OR-Split with regulation
OR-Join with regulation
A eS-Net with concurrency