Mobile Ad Hoc Networks
Trade-Offs and Topology Control

6th Week

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CoNe Freiburg

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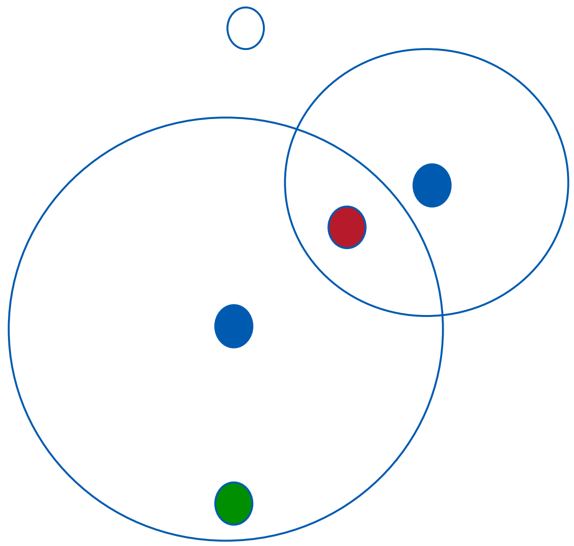
A Simple Physical Network Model

- **Homogenous Network of**
  - n radio stations $s_1,..,s_n$ on the plane
- **Radio transmission**
  - One frequency
  - Adjustable transmission range
    - Maximum range > maximum distance of radio stations
    - Inside the transmission area of sender: clear signal or radio interference
    - Outside: no signal
  - Packets of unit length



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The Routing Problem

- **Given:**
  - n points in the plane, $V=(v_1,..,v_n)$
    - representing mobile nodes of a mobile ad hoc network
  - the complete undirected graph G = (V,E) as possible communication network
    - representing a MANET where every connection can be established
- **Routing problem (multi-commodity flow problem):**
  - $f : V \times V \rightarrow \mathbf{N}$, where f(u,v) packets have to be sent from u to v, for all $u,v \in V$
  - Find a path for each packet of this routing problem in the complete graph
- **The union of all path systems is called the Link Network or Communication Network**

Formal Definition of Interference

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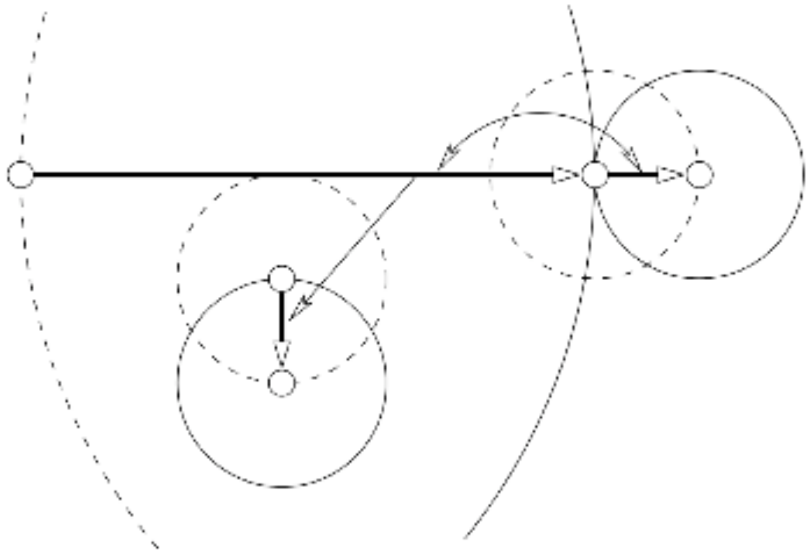
- Let $D_r(u)$ the disk of radius u with center u in the plane
- Define for an edge e={u,v} $D(e) = D_r(u) \cup D_r(v)$



- The set of edges interfering with an edge e = {u,v} of a communication network N is defined as:

$$\text{Int}(e) := \{e' \in E(N) \setminus \{e\} \mid u \in D(e') \text{ or } v \in D(e')\}$$

- The Interference Number of an edge is given by |Int(e)|
- The Interference Number of the Network is max{|Int(e)} | e ∈ E}

Formal Definition of Congestion

- **The Congestion of an edge e is defined as:**

$$C_{\mathcal{P}}(e) := \ell(e) + \sum_{e' \in \mathrm{Int}(e)} \ell(e')$$

- **The Congestion of the path system P is defined as**

$$C_{\mathcal{P}}(V) := \max_{e \in E_{\mathcal{P}}} \{C_{\mathcal{P}}(e)\}$$

- **The Dilation D(P) of a path system is the length of the longest path.**

Energy

- **The energy for transmission of a message can be modeled by a power over the distance d between sender and transceiver**
- **Two energy models:**
  - **Unit energy** accounts only the energy for upholding an edge
    - Idea: messages can be aggregated and sent as one packet

$$\text{U-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} |e|^2$$

  - **Flow Energy** Model: every message is counted separately

$$\text{F-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} \ell(e)|e|^2$$

A Measure for the Ugliness of Positions

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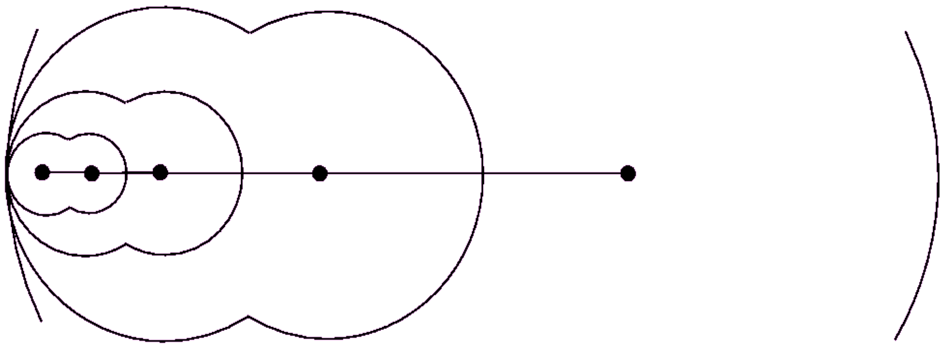
- **For a network G=(V,E) define the Diversity as**

$$g(V) := |\{m \mid \exists u,v \in V : \lfloor \log |u,v| = m \rfloor\}|$$

- **Properties of the diversity:**
  - g(V)=Ω(log *n*)
  - g(V)=O(*n*)



Congestion, Energy and Dilation

Congestion

Maximum number of packets interfering at an edge

$$C_{\mathcal{P}}(V) := \max_{e \in E_{\mathcal{P}}} \left\{ \ell(e) + \sum\nolimits_{e' \in \mathrm{Int}(e)} \ell(e') \right\} .$$

Energy

Sum of energy consumed in all routes

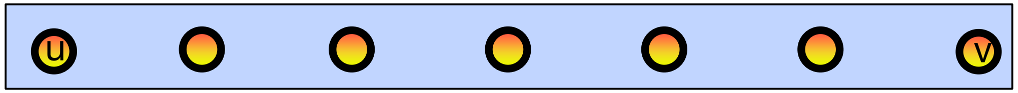
$$\mathrm{Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} \ell(e)|e|^2 .$$

Dilation

Maximum number of hops
(diameter of the network)

Energy versus Dilation

- **Is it possible to optimize energy and dilation at the same time?**
- **Scenario:**
  - n+1 equidistant nodes $u_0, ..., u_n$ on a line with coordinates 0,d/n, 2d/n,...,d



  - Demand: W packets from u_0 to u_n

- **Optimal path system for energy:**
  - send all packets over path $u_0, ..., u_n$
  - Dilation: n
  - $\text{Unit-Energy} = \sum_{i=1}^{n} \left(\frac{d}{n}\right)^2 = \frac{d^2}{n}$
  - $\text{Flow-Energy} = \sum_{i=1}^{n} W \left(\frac{d}{n}\right)^2 = \frac{d^2 W}{n}$

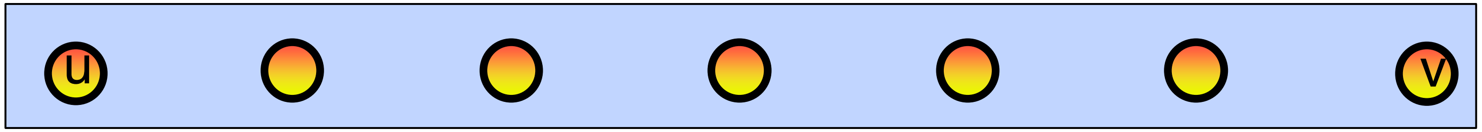
- **Optimal path system for dilation:**
  - send all packets over path u_0, u_n
  - Dilation: 1
  - $\text{Unit-Energy} = d^2$
  - $\text{Flow-Energy} = d^2 W$

- **Theorem: In this scenario we observe for all path systems:**

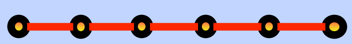
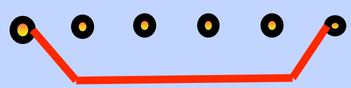
$$\textit{Unit-Energy} \times \textit{Dilation} \geq d^2$$

$$\textit{Flow-Energy} \times \textit{Dilation} \geq W d^2$$

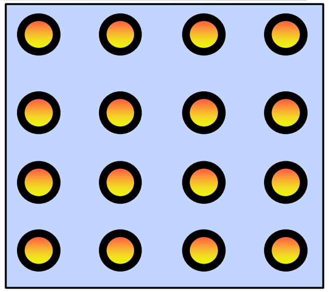
Tradeoff between Energy and Dilation



Demand of W packets between u and v

			any basic network
Energy E	$\frac{d^2W}{n}$	d^2W	$D \cdot E = \Omega(d^2W)$
Dilation D	n	1	

Congestion versus Dilation



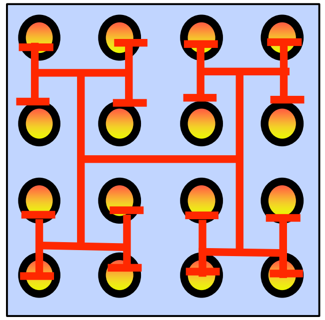
- **Is it possible to optimize congestion and dilation at the same time?**
- **Scenario:**
  - A $\sqrt{n} \times \sqrt{n}$ grid of n nodes (for a square number n)
  - Demand: W/n^2 packets between each pair of nodes
- **Optimal path system w.r.t. dilation**
  - send all packets directly from source to target
  - Dilation: 1
  - Congestion: $\Theta(W)$
    - if the distance from source to target is at least (3/4) n, then the communication disks cover the grid
    - So, a constant fraction of all W messages interfere with each other
- **Good path system w.r.t. congestion**
  - send all packets on the shortest path with unit steps
    - first horizontal and then vertical
  - Congestion: $O(W/\sqrt{n})$
    - On all horizontal lines at most $O(W/\sqrt{n})$ packets can interfere each other
    - Influence of horizontal on vertical lines increases the congestion by at most a factor of 2.
  - Dilation: $\sqrt{n}$

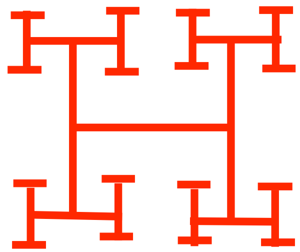
Congestion versus Dilation

- **Is it possible to optimize congestion and dilation at the same time?**
- **Scenario:**
  - A $\sqrt{n} \times \sqrt{n}$ grid of n nodes (for a square number n)
  - Demand: W/n^2 packets between each pair of nodes
- **Good path system w.r.t. dilation**
  - Build a spanning tree in H-Layout with diameter O(log n)
  - Dilation: O(log n)
  - Congestion: Θ(W (log n))
- **Theorem**
  - For any path system in this scenario we observe

$$\text{Congestion} \times \text{Dilation} = \Omega(W)$$

- **Proof strategy:**
  - Vertically split the square into three equal rectangles
  - Consider only 1/9 of the traffic from the leftmost to the rightmost rectangle
  - Define the communication load of an area
  - Proof that the communication load is a lower bound for congestion
  - Minimize the communication load for a given dilation between the rectangles





Trade-Off between Dilation and Congestion

Theorem

Given the grid vertex set G_n in d-dimensional space ($d \in \{2,3\}$) with traffic W then for every path system $\mathcal{P}$ the following trade-off between dilation $D_{\mathcal{P}}(G_n)$ and congestion $C_{\mathcal{P}}(G_n)$ exists:

$$C_{\mathcal{P}}(G_n) \cdot (D_{\mathcal{P}}(G_n))^{d-1} \geq \Omega(W) \ .$$

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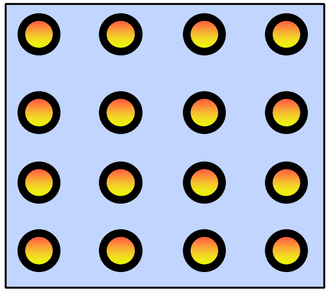
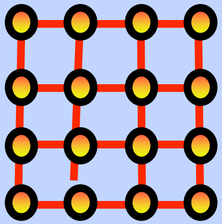
Tradeoff between Dilation and Congestion

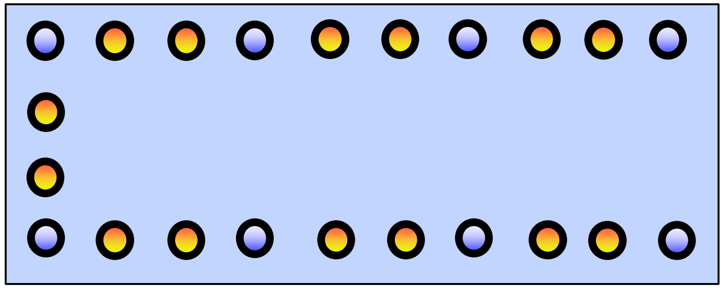
- n sites on a grid
- Between each pair of sites demand of W/n^2 packets



	Grid 	Direct	any basic network
Dilation	$\sqrt{n}$	1	$C \cdot D = \Omega(W)$
Congestion	$\frac{W}{\sqrt{n}}$	W	

Congestion versus Energy

- **Is it possible to optimize congestion and energy at the same time?**
- **Scenario:**
  - The vertex set $U_{\alpha,n}$ for a $\in [0,0.5]$ consists of two horizontal parallel line graphs line graphs with n^{α} blue nodes on each line
  - Neighbored (and opposing) blue vertices have distance Δ/n^{α}
- **Vertical pairs of opposing vertices of the line graphs have demand W/n^{α}**
- **Then, there are n other nodes equdistantly placed between the blue nodes with distance Δ/n vertices are equidistantly placed between the blue nodes**
- **Best path system w.r.t. Congestion**
  - One hop communication between blue nodes: Congestion: $O(W/n^{\alpha})$
  - Unit-Energy: : $\Omega(\Delta^2\, n^{-\alpha})$
  - Flow-Energy: $\Omega(W\, \Delta^2\, n^{-\alpha})$



- **Best path w.r.t Energy:**
  - U-shaped paths
  - Unit-Energy: $O(\Delta^2\, n^{-1})$
  - Flow-Energy: $O(\Delta^2\, n^{-1}\, W)$
  - Congestion: $\Omega(W)$
- **Choose α=1/3**

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Energy and Congestion are incompatible

Theorem 1 *There exists a vertex set V with a path system minimizing congestion to C^*, and another path system optimizing unit energy by U-Energy* and minimal flow energy by F-Energy* such we have for any path system $\mathcal{P}$ on this vertex set V we have*

$$\begin{aligned} C_{\mathcal{P}}(V) &\geq \Omega(n^{1/3} C^*) \quad or \\ U\text{-}Energy_{\mathcal{P}}(V) &\geq \Omega(n^{1/3} U\text{-}Energy^*)\ , \\ C_{\mathcal{P}}(V) &\geq \Omega(n^{1/3} C^*) \quad or \\ F\text{-}Energy_{\mathcal{P}}(V) &\geq \Omega(n^{1/3} F\text{-}Energy^*)\ . \end{aligned}$$

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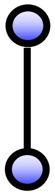
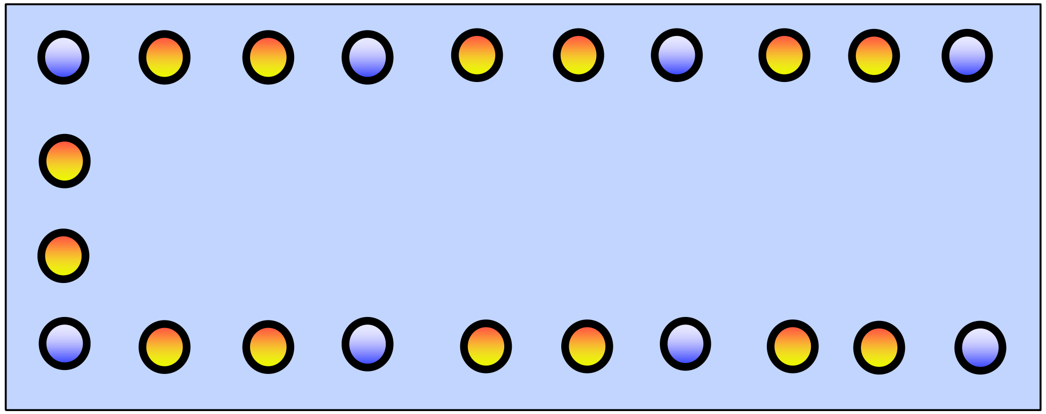
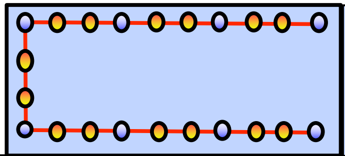
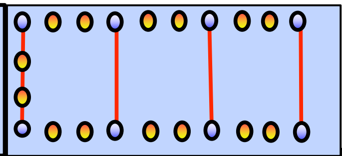
- $n^{1/3}$ blue sites 
- One packet demand between all vertical pairs of blue sites 



			any link network	
Congestion	$n^{1/3}$	$C^* = O(1)$	$C \geq \Omega(n^{1/3}C^*)$	*either*
Energy	$E^*=O(1/n)$	$O(1/n^{2/3})$	*or*	$E \geq \Omega(n^{1/3}E^*)$

Topology Control in Wireless Networks

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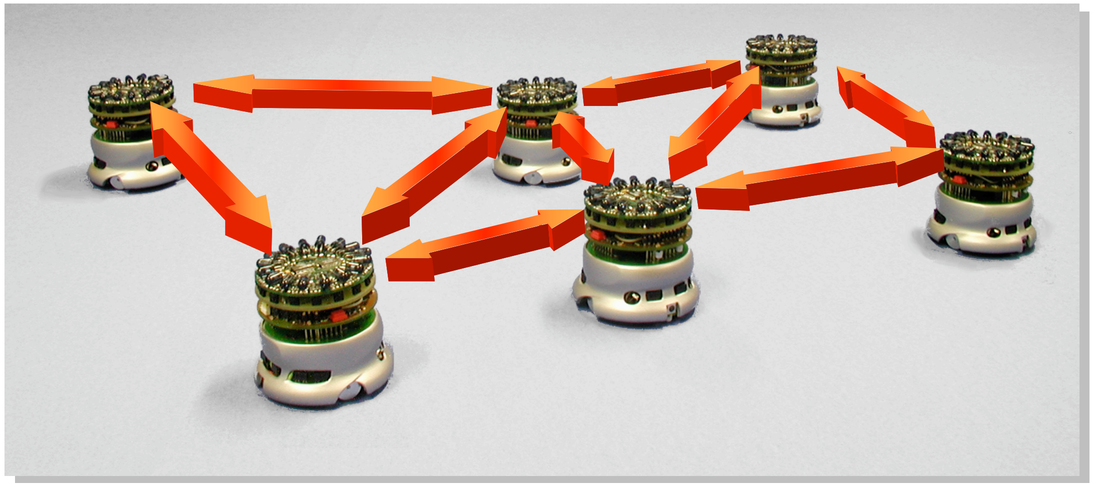
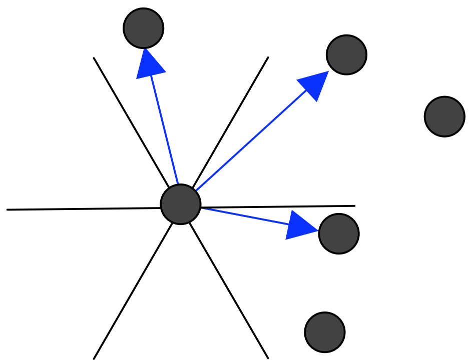
- Topology control: establish and maintain links
- Routing is based on the network topology
- Geometric spanners as network topologies



Yao-Graph

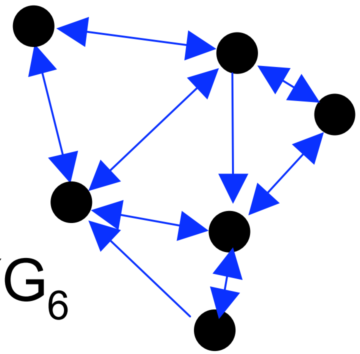
Yao-Graph

- Choose nearest neighbor in each sector
- c-spanner, i.e. constant stretch-factor
- distributed construction



c-Spanner [Chew86]

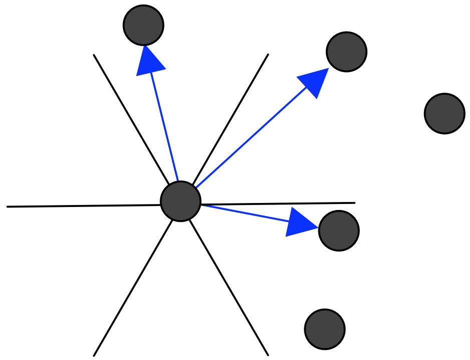
c-spanner:
for every pair of nodes u,v
there exists a path P s.t. $||P|| \leq c \cdot ||u,v||$

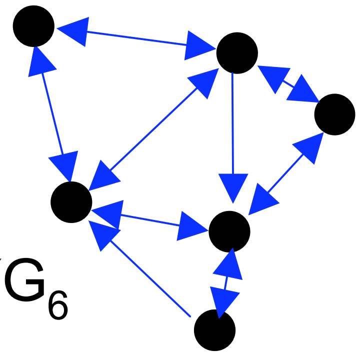


YG_6

Spanner Graphs and Yao-Graphs

- **Definition**
  - A c-Spanner is a graph where for every pair of nodes u,v there exists a path P s.t.
    $||P|| \leq c \cdot ||u,v||$.
- **Motivation:**
  - Short paths
  - Energy optimal paths
- **Example of a Spanner-Graph:**
  - Yao-graph
- **Defintion Yao-Graph (Theta-Graph)**
  - Given a node set V
  - Define for each node k sectors $S_1(u)$, $S_2(u)$, ..., $S_k(u)$ of angle $\theta = 2\pi/k$ with same orientation
  - The Yao-Graph consists of all edges
    $E = (u,v \mid \text{exists } i \in \{1,..,k\}: v \in S_i(u) \text{ and for all } v' \in S_i(u): ||u,v'|| \geq ||u,v|| \}$

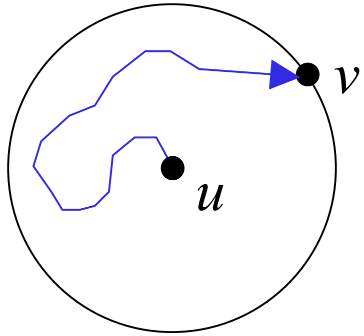




YG_6

Weaker Spanning

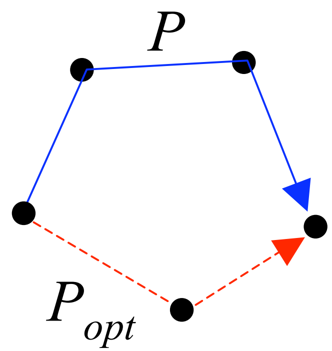
Weak-Spanner [FMS97]



for every pair of nodes u,v
exists a path inside the disk
C($u, c \cdot ||u,v||$)

...sufficient for allowing routing which approximates minimal congestions by a factor of O(Int(G) g(V))
[Meyer auf der Heide, S, Volbert, Grünewald 02]

Power-Spanner [LWW01, GLSV02]



for every pair of nodes u,v
exists path P s.t. $|P| \leq c \cdot |P_{opt}|$
$|P| = \Sigma\, |v_i, v_{i+1}|^d$

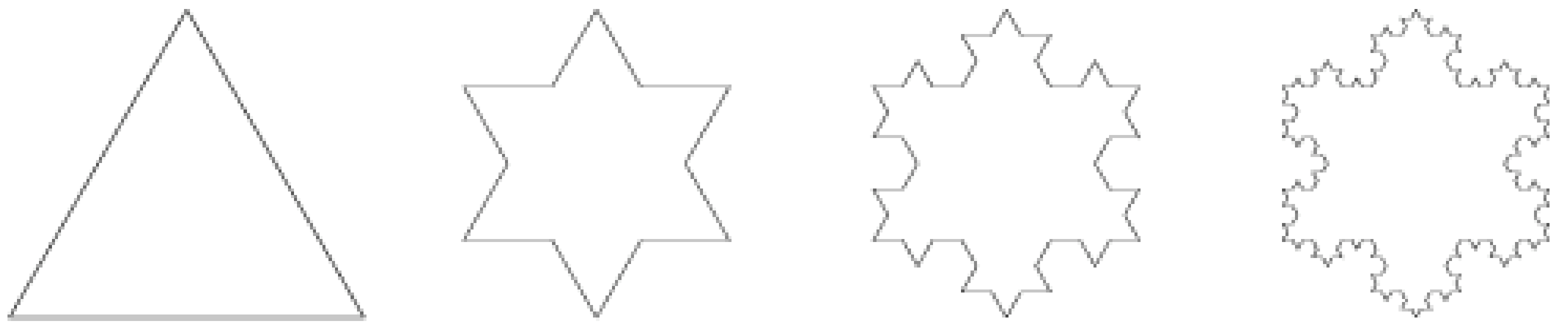
...approximates energy-optimal path-system

Spanners, Weak Spanners, Power Spanners

- **Theorem**
  - Every c-Spanner is a c-weak spanner.
- **Theorem**
  - Every c-weak-Spanner is a c’-power Spanner when $d \geq 2$.
- **Proof:**
  - straightforward for $d>2$
  - involved construction for $d=2$

The Koch Curve is not a Spanner

- **Koch-Curves: Koch 0, Koch 1, Koch 2 ,...**



- **Theorem**
  - The Koch Curve is not a c-Spanner
- **Theorem**
  - The Koch Curve is a weak 1-Spanner.

Yao-Family

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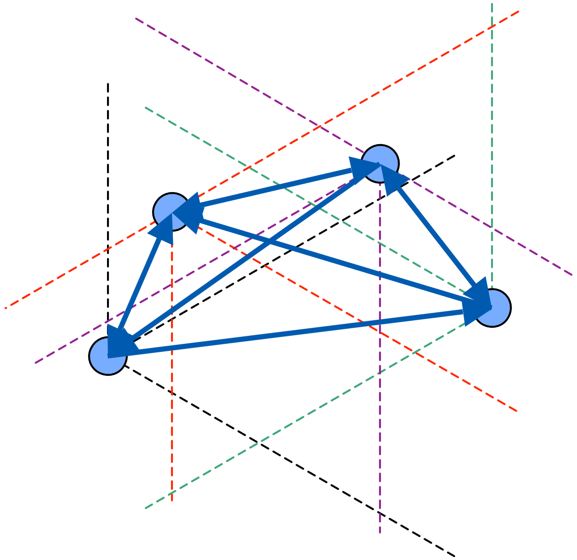
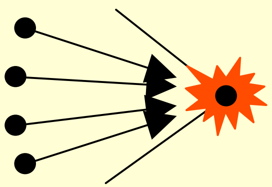


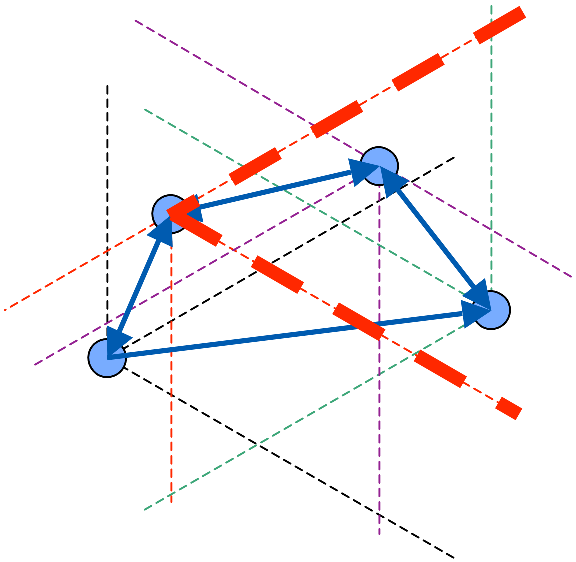
Yao-Graph

Spanner

Disadvantage:
Unbounded in-degree



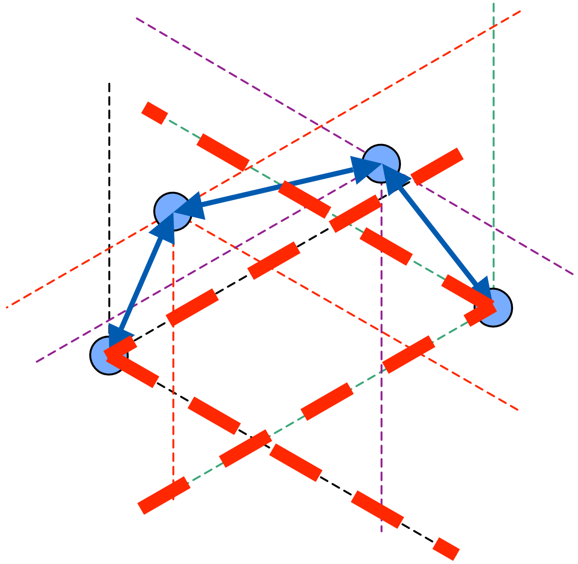
Interferences !



⊇ **SparsY**

Sparsified Yao-Graph

use only the shortest ingoing edges
weak- & power-Spanner,
constant in-degree

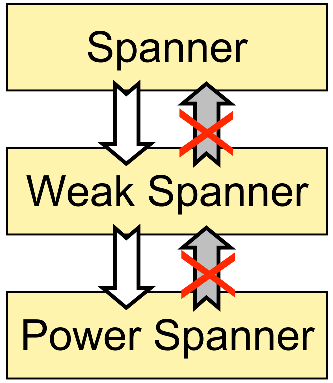


⊇ **SymmY**

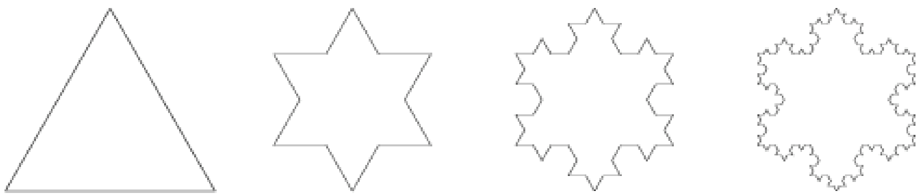
Symmetric Yao-Graph

only symmetric edges
not a spanner,
nor weak spanner,
yet power-spanner

Spanner, Weak Spanner, Power Spanner



- **Every c-Spanner is a weak c-Spanner**
- **Every c-Spanner is a (c^d,d)-Power Spanner**
- **Every weak c-Spanner is a (c',d)-Power Spanner for d≥2**
- **There are weak Spanners that are no Spanners**
  (e.g. the Koch Curve is no c-Spanner but a weak 1-Spanner)
- **There are Power Spanners that are no Weak Spanners**



Power Spanners and Weak Spanners

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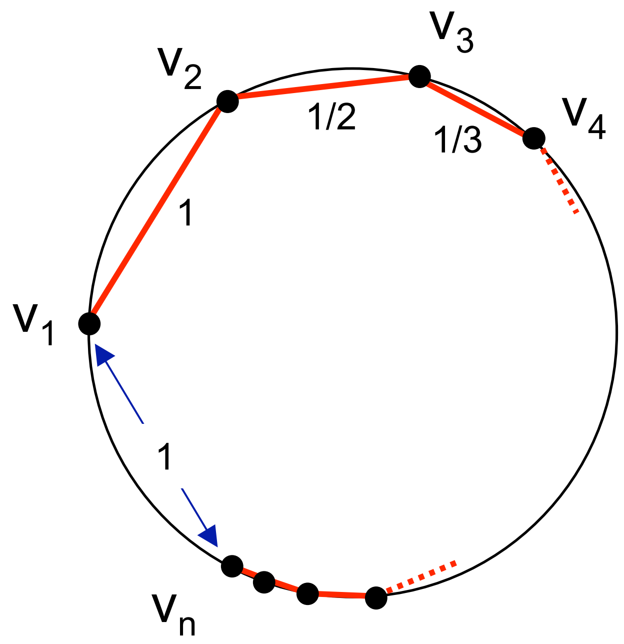
- Place n nodes $v_1,\ldots,v_n$ on a circle such that $|v_i - v_{i+1}| = 1/i$
- The circle is scaled such that $|v_1 - v_n| = 1$
- Consider $G = (V,E)$ with $V = \{v_1,\ldots,v_n\}$ and $E = \{(v_i,v_{i+1}) \mid i=1,\ldots,n-1\}$
- G is a (c,d)-Power Spanner:

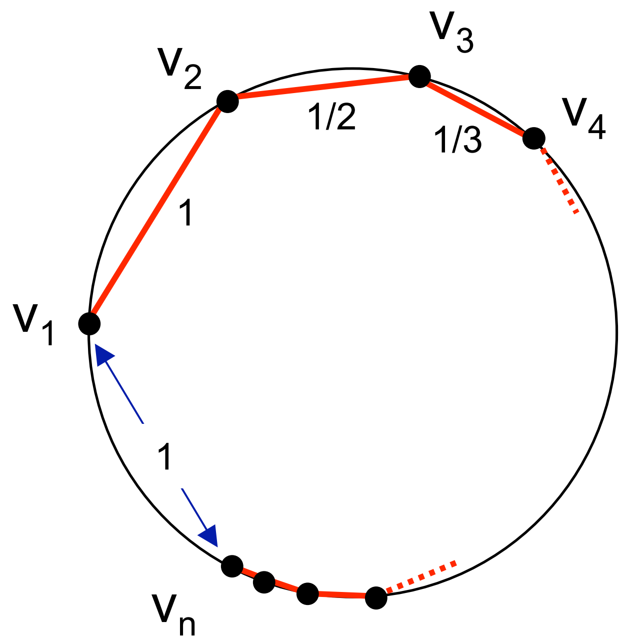


$$\text{Energy}(P) = \sum_{i=1}^{n-1} (1/i)^d \le \sum_{i=1}^{\infty} (1/i)^d = \mathcal{O}(1) \qquad (d>1)$$

$$\forall c > -1: \sum_{i=1}^{n} i^c = \Theta(n^{c+1}) \qquad \forall c < -1: \sum_{i=1}^{n} i^c = \mathcal{O}(1)$$

Power Spanners and Weak Spanners

- $|v_i - v_{i+1}| = 1/i$ and $|v_1 - v_n| = 1$
- $G = (V,E)$ with $V = \{v_1,\ldots,v_n\}$ and $E = \{(v_i,v_{i+1}) \mid i=1,\ldots,n-1\}$
- G is a (c,d)-Power Spanner
- G is not a Weak Spanner:
  Radius of the circle depends on the Euclidean length of the chain:



$$\sum_{i=1}^{n-1} \frac{1}{i} = \Theta(\log n)$$

$$\ln n \leq \sum_{i=1}^{n} \frac{1}{i} \leq 1 + \ln n$$

The Yao-Family

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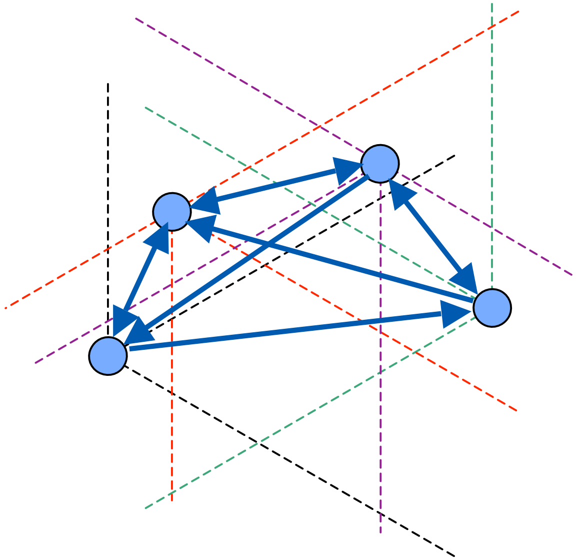
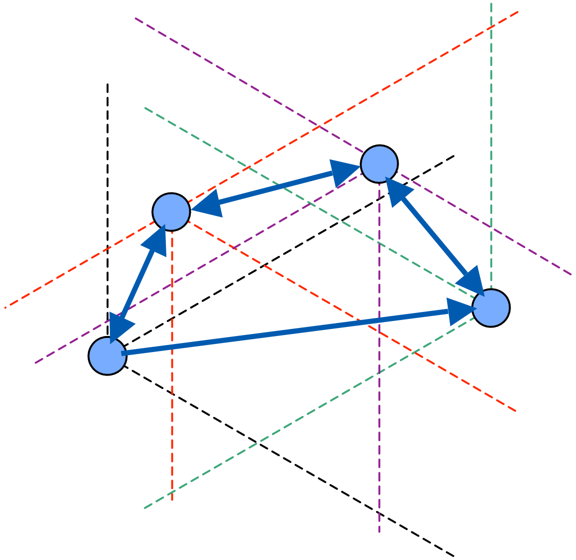


Yao-Graph

nearest neighbor
in each sector

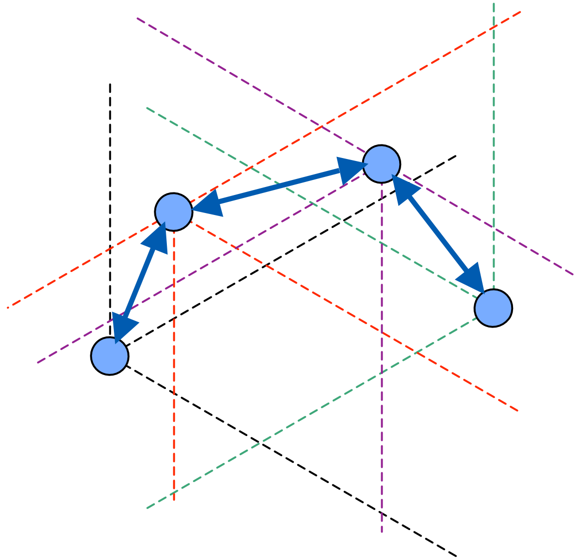
Spanner



⊇ **SparsY**
Sparsified Yao-Graph

use only the shortest
ingoing edges

weak- & power-Spanner,
constant in-degree



⊇ **SymmY**
Symmetric Yao-Graph

only symmetric edges

not a spanner,
nor weak spanner,
nor power-spanner

The Symmetric Yao Graph (SymmY)

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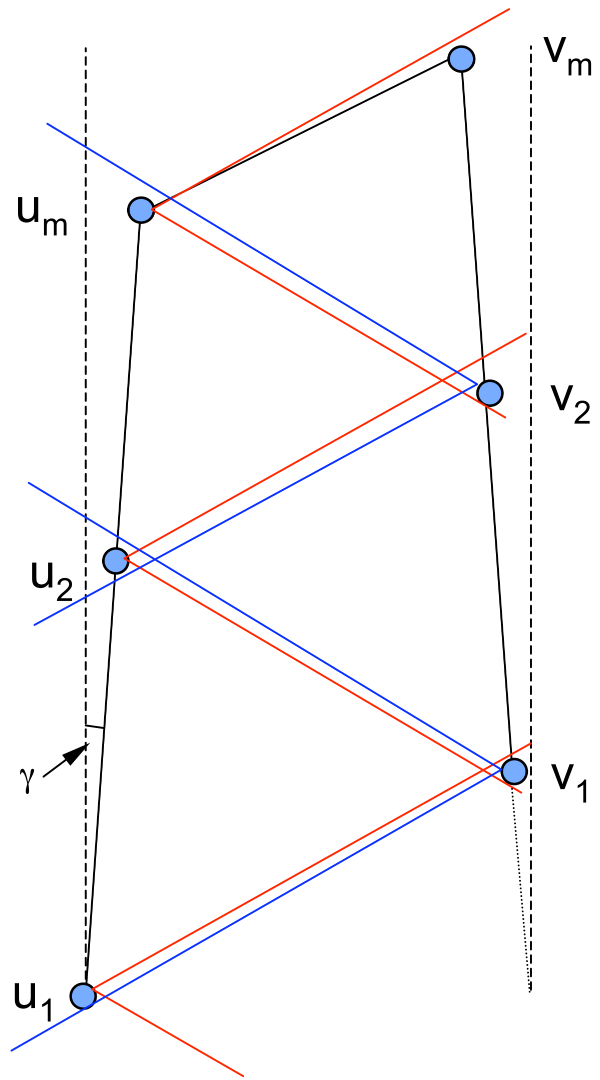
- **SymmY is not a c-Spanner**
- Worst case construction →



The Hierarchical Layer Graph (HLG)

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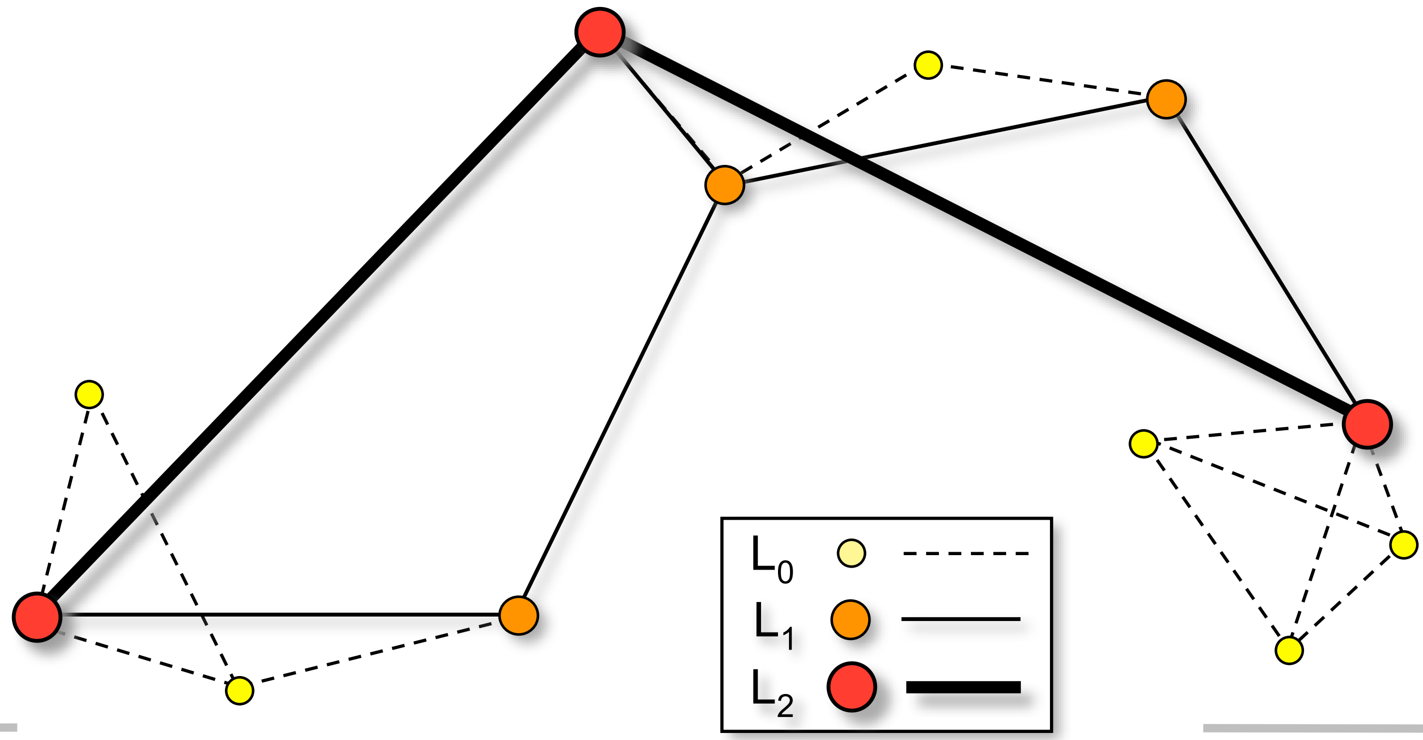
- **Basic Ideas:**
  - many short edges on lower layers → energy efficiency
  - few long edges on higher layers → connectivity
- layers **= range classes, assigned to power levels**



Construction of the HL Graph

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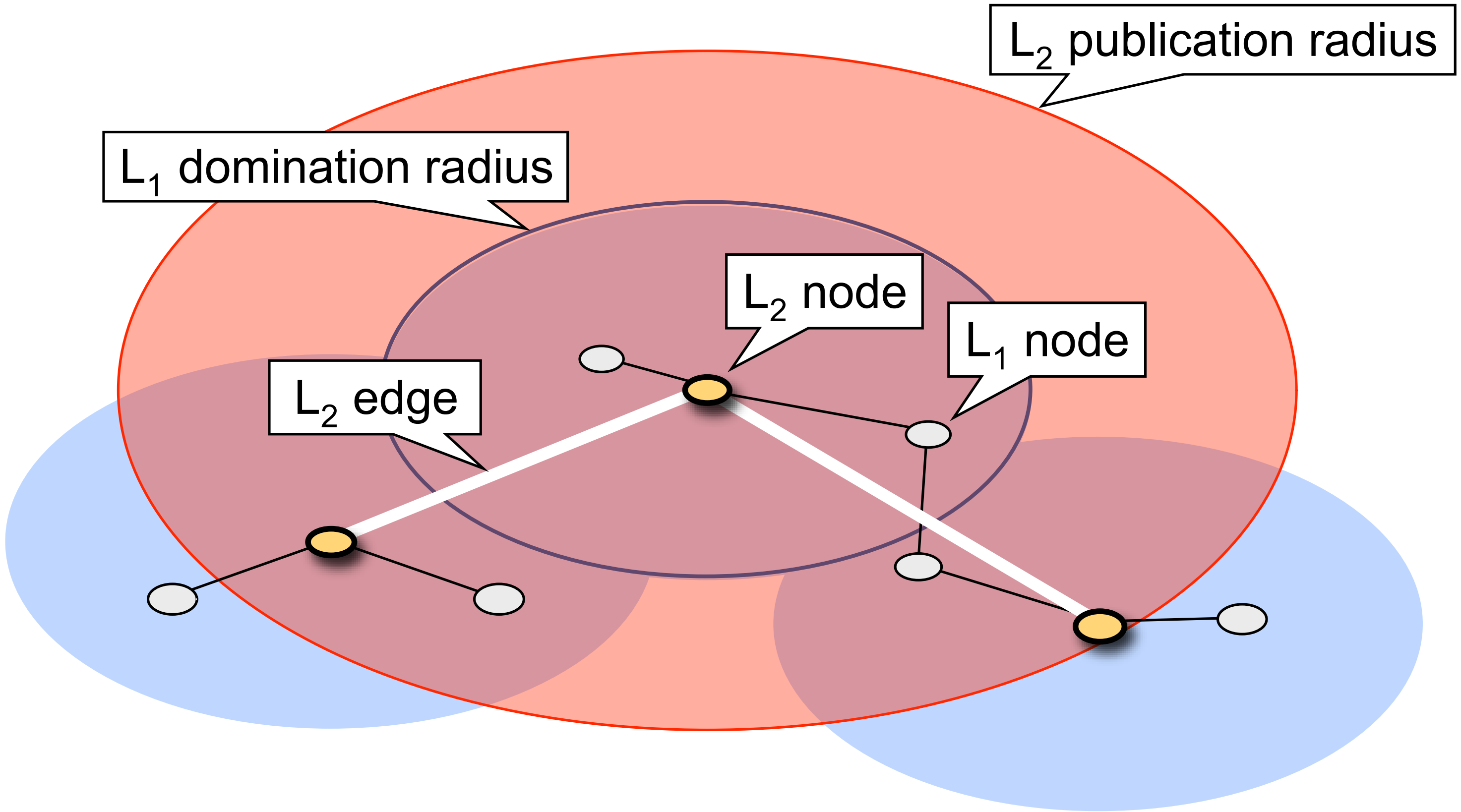
- node with the highest priority on layer 1 becomes L_2 node
  ...and dominates L_1 nodes
- L_2 node connects to other L_2 nodes



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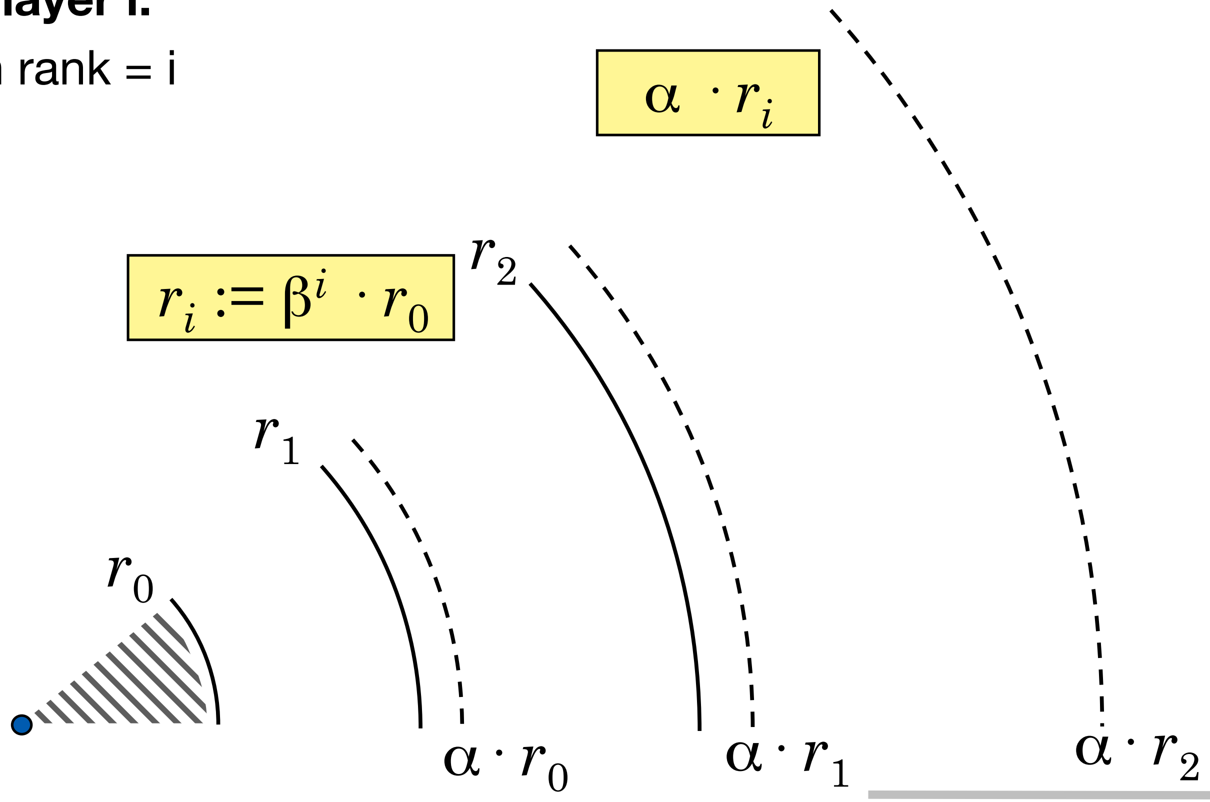
Radii of the HL Graph

- **definition based on parameters α and β**
- **r_0 := minimal node distance, rank := highest layer**
- domination radius **for layer i:**
  no other nodes with rank > i within this radius
- publication radius **for layer i:**
  edges to nodes with rank = i



Radii and Edges of the HL Graph

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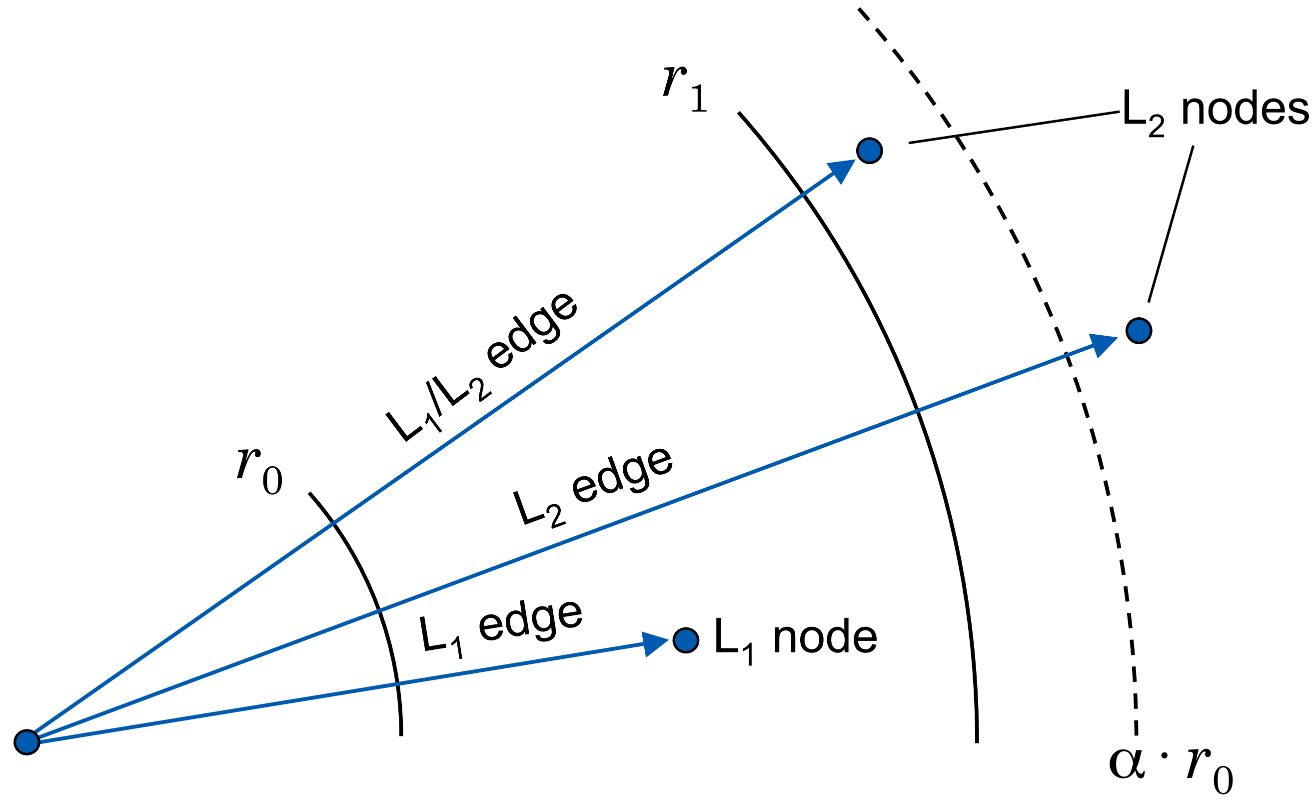
- layer-(i-1) publication radius > layer-i domination radius: $\alpha > \beta$
- layer-i edges are established in between



Properties of the HL Graph

- **The HL Graph is a c-Spanner, if $\alpha > 2\beta / (\beta-1)$**

- **The interference number of the HLG is bounded by O(g(V))**

  g(V) = Diversity of the node set V
  g(V) = O(log n) for nodes in random positions with high probability

- **A c-Spanner contains a path system with load $O(g(V) \cdot C^*)$**

  C* = congestion of the congestion-optimal path system

- **The HLG contains a path system P with congestion $O(g(V)^2 \cdot C^*)$**

  i.e. P approximates the congestion-optimal path system by a factor of $O(\log^2 n)$ for nodes in general position

Thank you!

CoNe
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