Mobile Ad Hoc Networks

Network Coding and Xors in the Air

7th Week

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CoNe Freiburg

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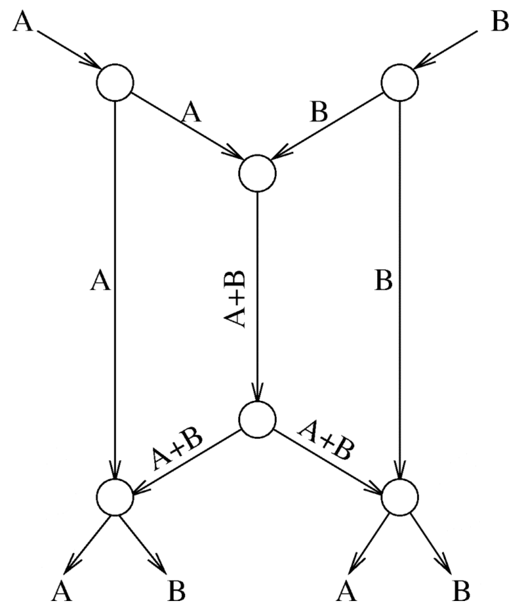
Network Coding

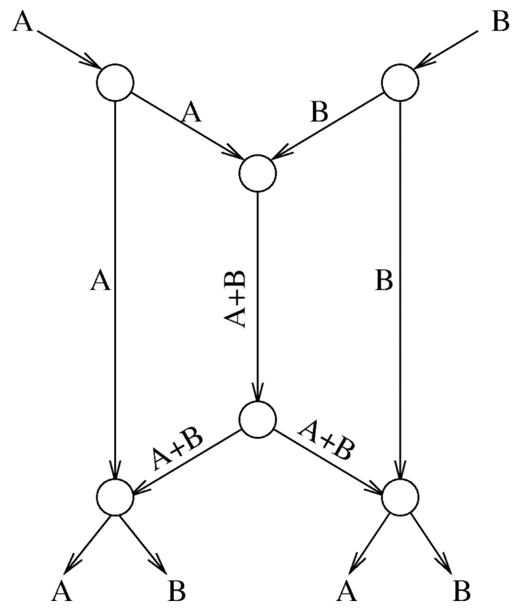
- **R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network Information Flow", *(IEEE Transactions on Information Theory*, IT-46, pp. 1204-1216, 2000)**
- **Example:**
  - Bits A and B need to be transfered
  - Every link transmits only a bit
  - If the bits must be unchanged then
    - A and B can be received either on the right or on the left side
  - Solution: Compute Xor A+B in the middle link and both sides get A and B



Network Coding and Flow

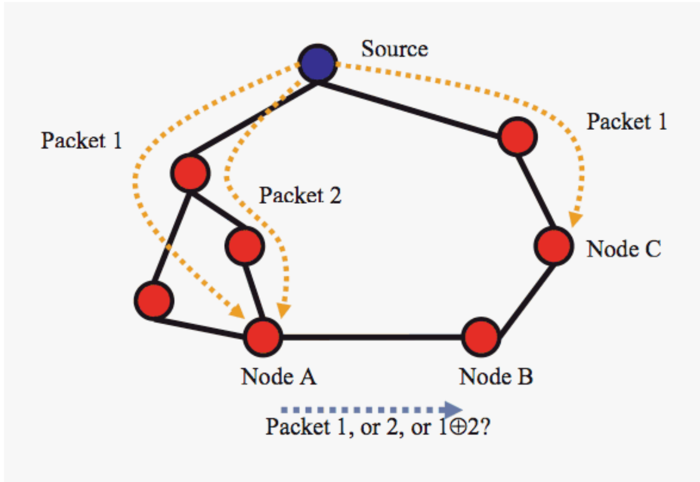
- **R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network Information Flow", *(IEEE Transactions on Information Theory*, IT-46, pp. 1204-1216, 2000)**

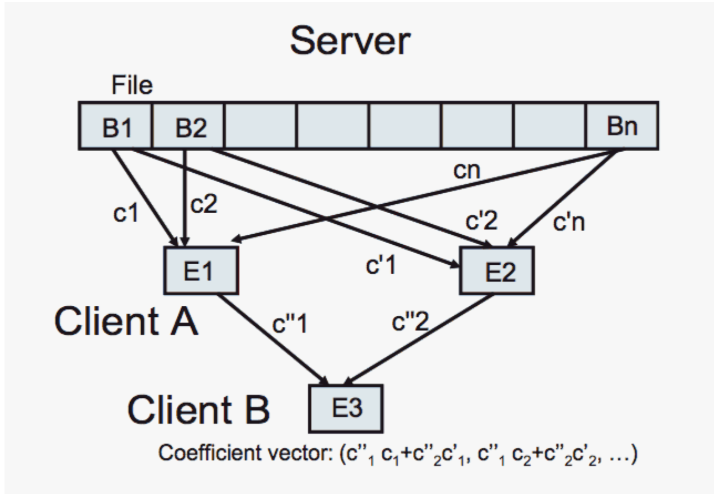
- **Theorem [Ahlswede et al.]**
  - There is a network code for each graph such that each target nodes receives as much information as the maximal flow problem for each target allows



Practical Network Coding in Peer-to-Peer Networks

- **Christos Gkantsidis, Pablo Rodriguez Rodriguez, 2005**
- **Goal**
  - Overcome the coupon collector problem for partitioning of data
    - A message of m frames can be received if the sum of the m received encoded frames is at least m
  - Optimal transmission of files w.r.t the available bandwidth
- **Method**
  - Use linear combinations of the frames of the message
    - Send combination with the corresponding variables
  - Recombine transmitted frames in intermediate stations
  - Receivers collect the linar combinations
  - Use matrix inverse of the parameters to reconstruct the original message





Encoding and Decoding

- **Original message frames: x_1, x_2, ..., x**
- **Encoded frames: y_1,y_2,...,y_m**
- **Random variables r_{ij}**

$$(r_{i1} r_{i2} \ldots r_{im}) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = y_i$$

- **Hence**

$$\begin{pmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mm} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

- **If the matrix (r_{ij}) is invertable, then we have**

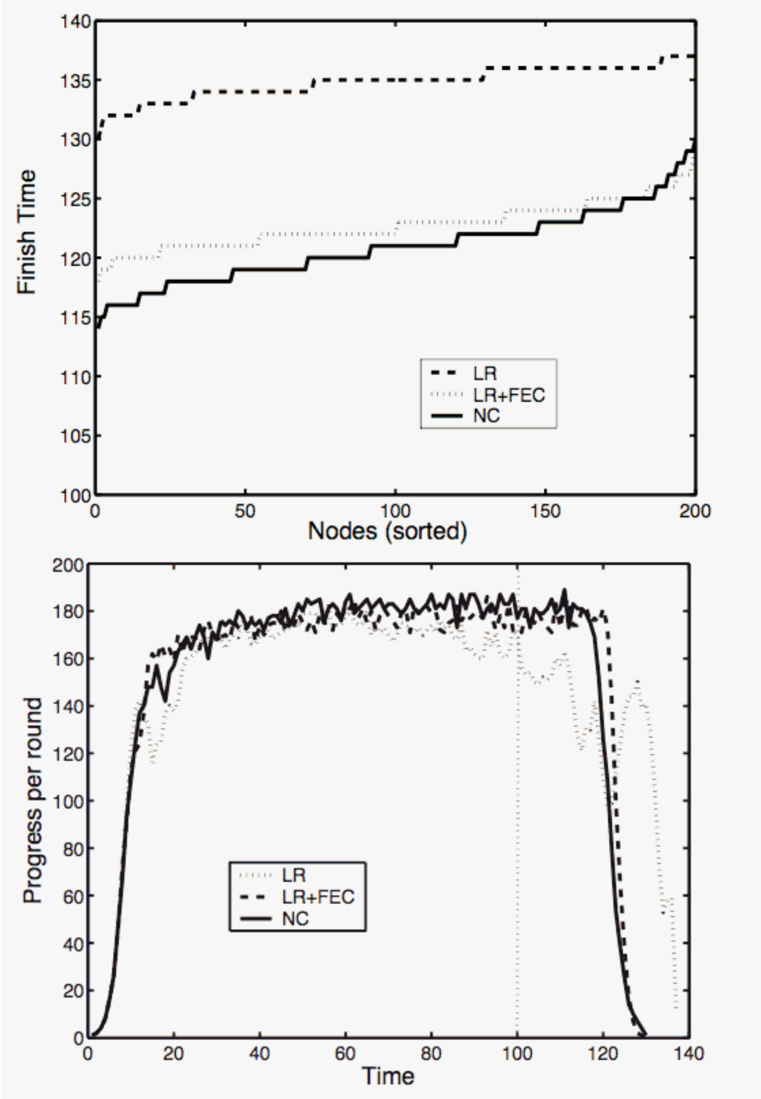
$$\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mm} \end{pmatrix}^{-1} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

On Inverting a Random Matrix

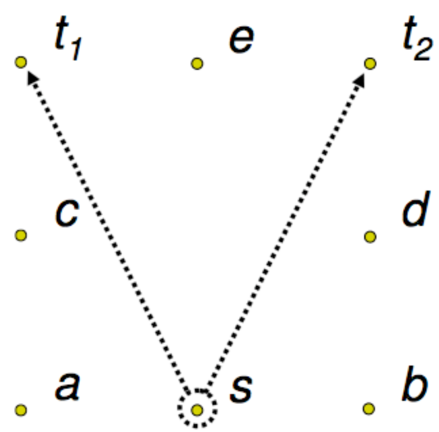
- **Theorem**
  - If the numbers of a m x m random matrix are chosen uniformly and independently from a finite field of size b, then the random matrix can be inverted with probability of at least
  - $$1 - \sum_{i=1}^{m} \frac{1}{b^i}$$
- **Idea: Choose finite field GF[2^8]**
  - Computation with bytes is very efficient
  - The success probability is at least 0.99
  - In the error case an additional frame gives again a success probability of at least 0.99

Speed of Network Coding in Peer-to-Peer-Networks

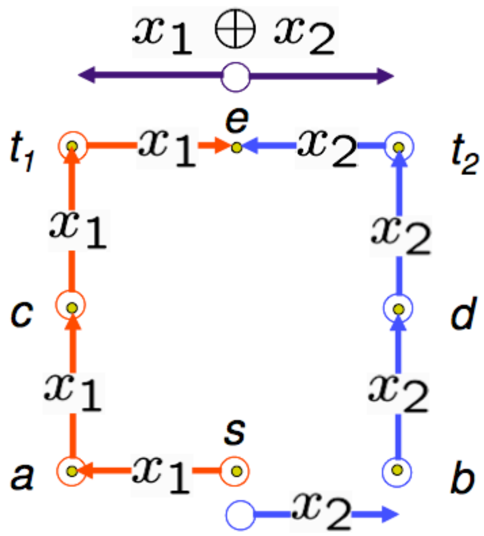
- **Comparison**
  - Network-Coding (NC) versus
  - Local-Rarest (LR) and
  - Local-Rarest+Forward-Error-Correction (LR+FEC)



- **Minimum-Energy Multicast in Mobile Ad hoc Networks using Network Coding, Yunnan Wu, Philip A. Chou, Sun-Yuan Kung, 2006**
- **Multicast: Send message from one node to a dedicated set**
- **Example:**
  - Traditional cost: 5 energy units for 1 message
  - With network coding: 9 energy units for 2 messages



(a)



(b)

Multicasting in Ad Hoc Networks

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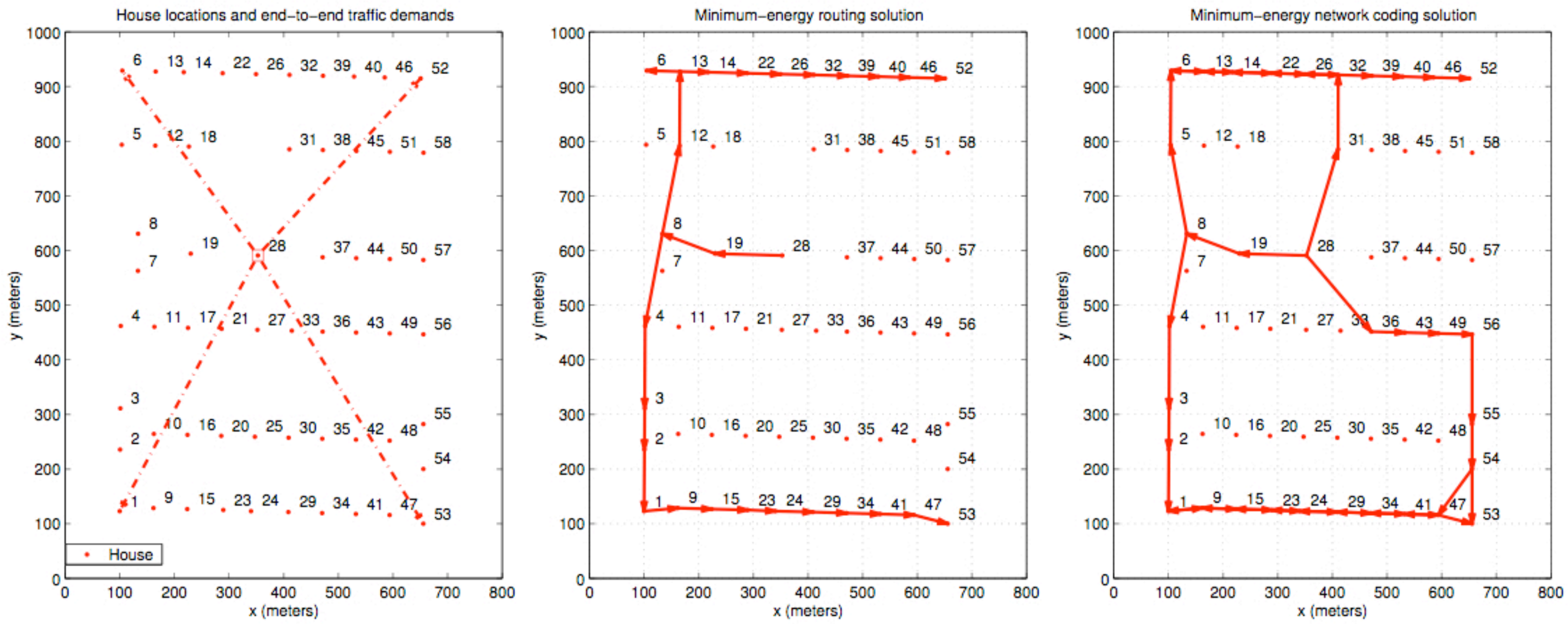
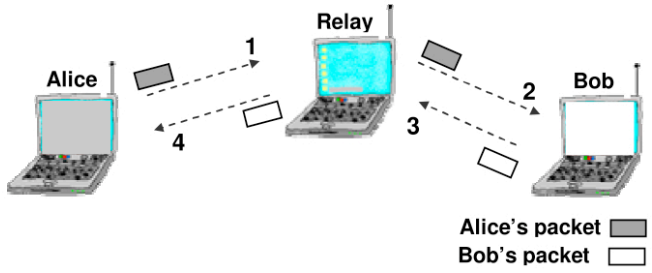
- **Minimum-Energy Multicast in Mobile Ad hoc Networks using Network Coding, Yunnan Wu, Philip A. Chou, Sun-Yuan Kung, 2006**
- **Solving minimal energy multicasting is NP-hard**
  - Problem: Solve an integer linear optimization problem
- **With network coding the maximum throughput can be found in polynomial time**
  - Solve linear optimization problem, i.e. a flow problem



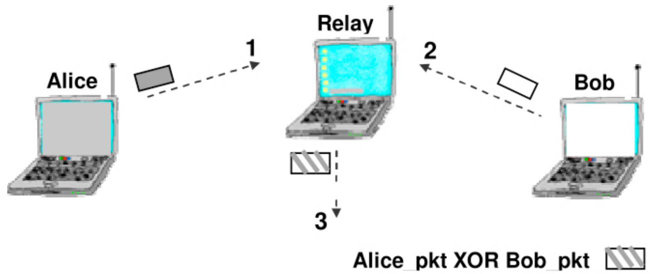
(a) (b) (c)

XOrs in the Air

- **XORs in the Air: Practical Wireless Network Coding, Sachin Katti Hariharan Rahul, Wenjun Hu, Katabi, Muriel Médard, Jon Crowcroft**
- **Problem:**
  - Maximize throughput in an ad hoc network
  - Multihop messages lead to interferences
- **Example**
  - Traditional: 4 messages to deliver a message from Alice to Bob and from B
  - Network Coding: 3 messages



(a) Current Approach



(b) COPE

Components of COPE

- **Opportunistic Listening**
  - Get maximum context for decoding messages
- **Opportunistic Coding**
  - „The key question is what packets to code together to maximize throughput. A node may have multiple options, but it should aim to maximize the number of native packets delivered in a single transmission, while ensuring that each intended nexthop has enough information to decode its native packet.“
- **Learning Neighbor State**
  - Each node announces the packets it has received
  - Each node also guesses the packets a neighbor could have received

Opportunistic Coding

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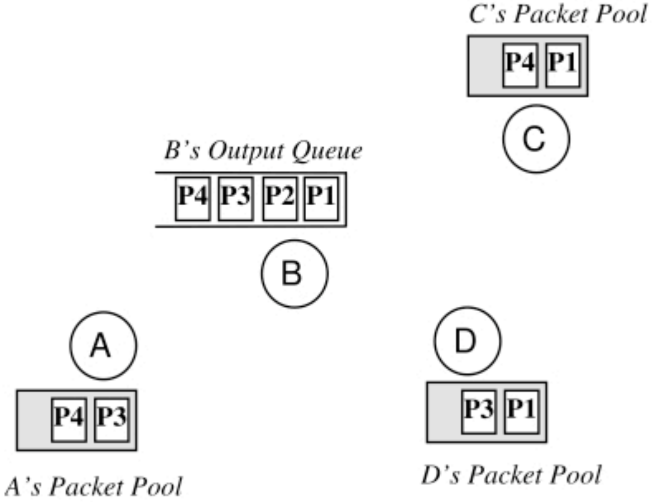


(a) B can code packets it wants to send

Packets in B's Queue	Next Hop
P1	→ A
P2	→ C
P3	→ C
P4	→ D

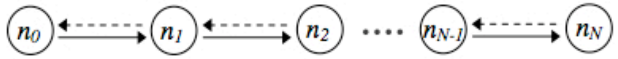
(b) Nexthops of packets in B's queue

Coding Option	Is it good?
P1 + P2	Bad Coding (C can decode but A can't)
P1 + P3	Better Coding (Both A and C can decode)
P1 + P3 + P4	Best Coding (Nodes A, C, and D can decode)

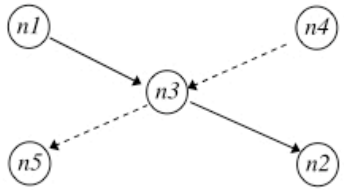
(c) Possible coding options

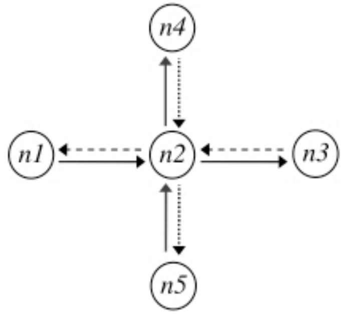
Theoretical Gains

- **Coding Gain:**
  - Number of messages saved because of network coding
- **Coding+MAC Gain:**
  - Intermediate routers forming a bottleneck further delay the medium access
  - Using COPE an additional speedup occurs



(a) Chain topology; 2 flows in reverse directions.



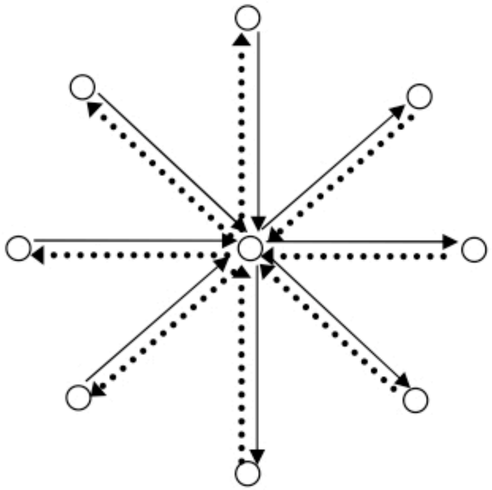


(b) "X" topology 2 flows intersecting at n_2.

(c) Cross topology 4 flows intersecting at n_2

Topology	Coding Gain	Coding+MAC Gain
Alice-and-Bob	1.33	2
"X"	1.33	2
Cross	1.6	4
Infinite Chain	2	2
Infinite Wheel	2	∞

Table 2—Theoretical gains for a few basic topologies.



(d) Wheel topology; many flows intersecting at the center node.

Summary Network Coding

- **Network Coding can help to**
  - increase traffic throughput in Ad Hoc Networks
    - COPE (in the absence of hidden terminal)
  - decrease energy consumption in multicast
  - increase robustness and reduce the error rate
  - increase throughput in Peer-to-Peer Networks
  - increase throughput in Wireless Sensor Networks
- **Many Network Coding schemes suffer from the complexity of inverting large matrices and introduce a delay for decoding**
- **COPE is an exemption it is efficient and without delay**

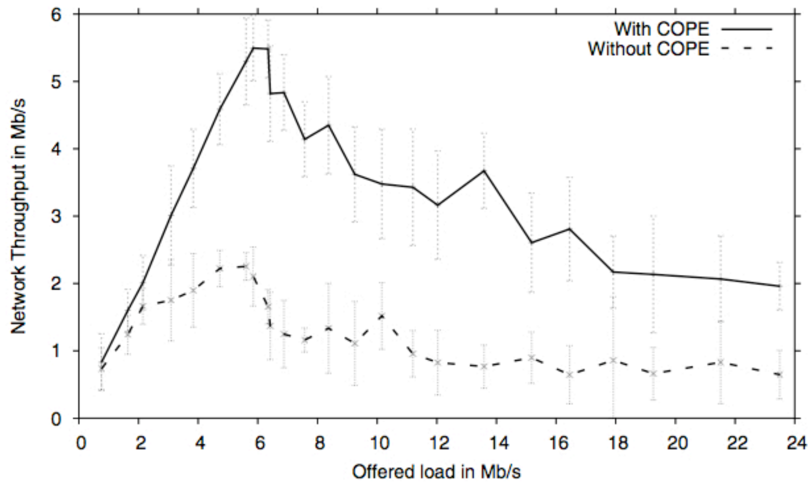


Figure 12—COPE can provide a several-fold (*3-4x*) increase in the throughput of wireless Ad hoc networks. Results are for UDP flows with randomly picked source-destination pairs, Poisson arrivals, and heavy-tail size distribution.

Thank you!

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