Exercise No. 1
Peer-To-Peer Networks
Summer 2008

Exercise 1 Graph Topologies
Consider the following graph topologies for \( n \) nodes.

1) ring

2) balanced binary tree
   (for \( n = 2^k, k \in \mathbb{N} \))

3) two-dimensional torus
   (for \( n = k^2, k \in \mathbb{N} \))

4) hypercube (for \( n = 2^k, k \in \mathbb{N} \))

5) complete graph

6) star

Indicate for each of the graph topologies the following parameters and sort them according to their asymptotical order.

a) maximal degree of a node

b) amount of edges

c) diameter of the graph

d) maximal amount of nodes that can be reached in \( d \) steps starting in one node

e) minimal amount of nodes that have to be removed such that the graph is no longer connected
Exercise 2  Hash-Functions

Given is a hash-table with \( n \) slots \( S_i, i \in \{0, 1, \ldots, n - 1\} \). A hash-function \( h \) is a mapping of given integer values to those slots, as uniform as possible. A collision occurs, if the hash-function \( h \) maps a value to a slot that is already occupied.

1. Choose a hash-function \( h_d \) for \( n = 7 \) and show how the numbers 4, 13, 25, 34, 46, and 55 are mapped to the slots \( S_i \).
   - How many collisions occur, how can they be treated?
   - Suppose a slot is removed, or a new slot \( S_7 \) is added. How does that change the mapping of \( h \)?

2. Suppose each number is mapped to a slot by a uniform random function \( h_r \).
   - Using the same collision treatment as above, how many collisions occur in expectation value?
   - What happens when slots are removed/added?
   - What advantage has \( h_r \) compared to \( h_d \) if the slots change? Also think about \( n \) being much larger than the number of values to store.
   - There is a serious disadvantage of \( h_r \). What is it?