Peer-to-Peer Networks

8. Distance-Halving

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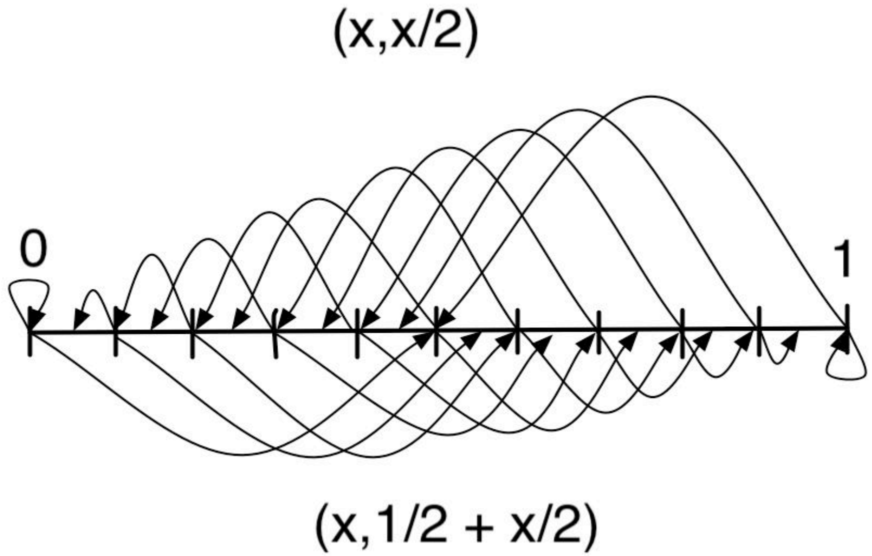
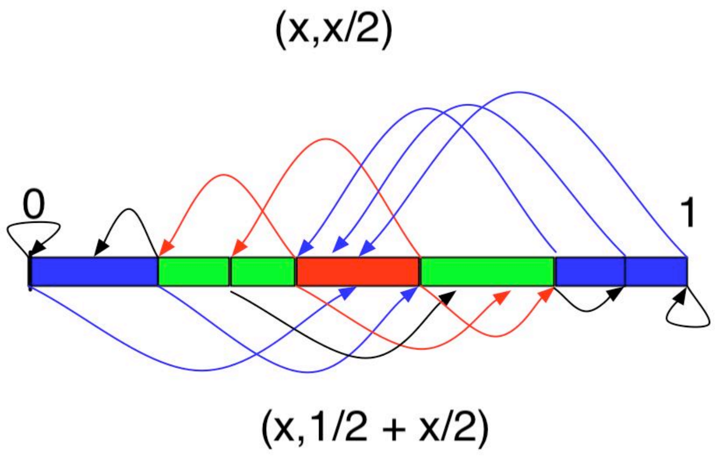
Continuous Graphs

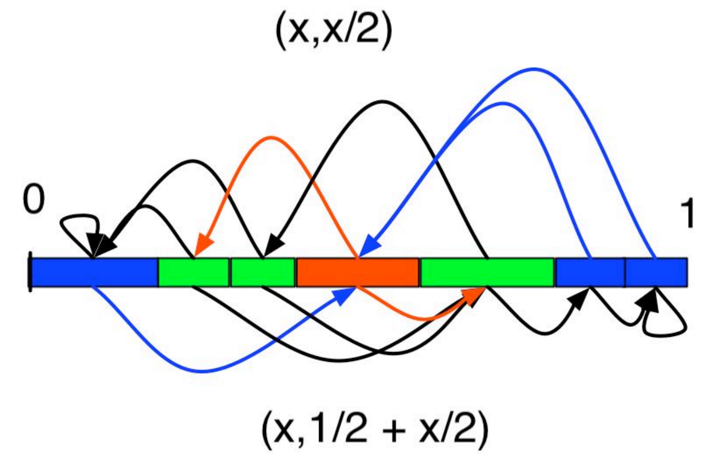
- Distance Halving
  - Moni Naor Udi Wieder, 2003
- Continuous graphs
  - Infinite set of nodes
  - Infinite set of edges
- Underlying graph
  - $x \in [0,1)$
  - Edges:
    - Left edges: (x,x/2)
    - Right edges: (x,1+x/2)
  - Plus opposing edges
    - (x/2,x)
    - (1+x/2,x)



Transition from the Continuous to the Discrete Case

- Consider discrete partitions of the continuous node set
- Insert edge from partition A to B
  - if there exists $x \in A$ and $y \in B$ such that (x,y) is an edge of the continuous graph
  - Partitions are constructed by halving larger partitions
- The degree is constant if
  - the ratio of the sizes of largest and smallest intervals are constant
- Can be achieved using a technique, called *multiple choice principle*



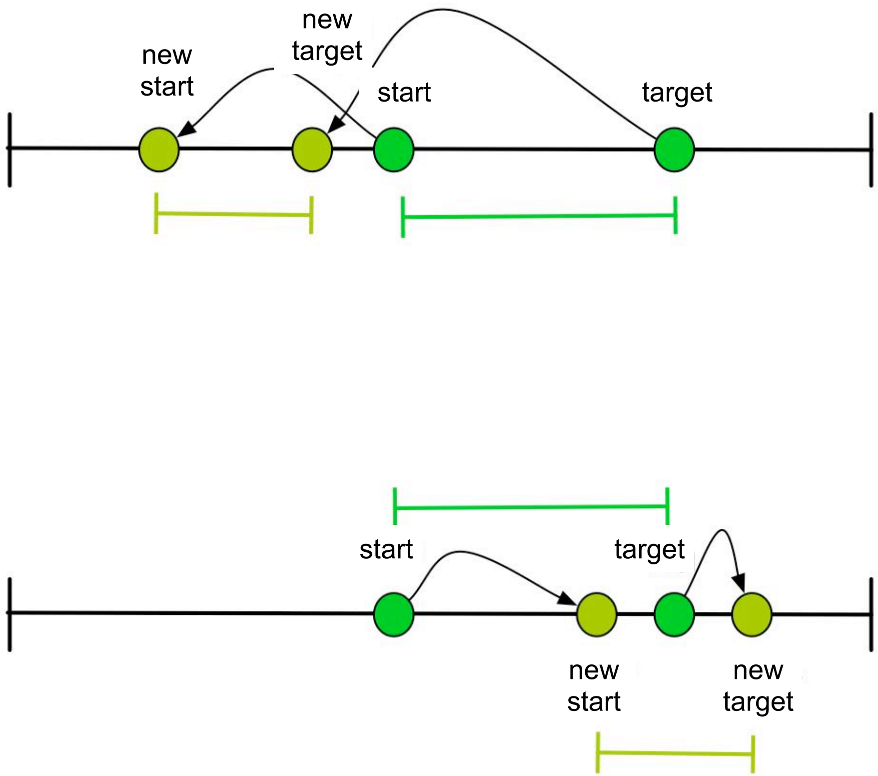


Features of Distance-Halving

- Peers correspond to partitions
- Links correspond to edges
  - In additions neighbored partitioned are linked
- In- and out-degree are constant
  - The largest partition has length of at most 2/n w.h.p.
  - The smallest partition has length of at least 1/(2n) w.h.p.
- The diameter is logarithmic

Lookup in Distance-Halving

- Using left edges the distance between start and target is halved
  - Follow the left edges for 2+log n hops
  - Then the target is a neighbored partition
  - Then the backward left edges are used to get to the target
- Works also using right edges
- Lemma
  - Lookup needs 2 log n + O(1) hops and messages



Congestion using Lookup

- Left and right edges can be used in any combination
- Analog to Valiant‘s routing result for relieving congestion on the hyper-cube one can shoe
  - The Congestion is bounded by O(log n) w.h.p.,
  - i.e. each peer has to deliver at most O(log n) packets if each peer is the start and target of at most one lookup

Insertion of Peers in Distance-Halving

- Use Principle of multiple choice, i.e.
  - test c log n random partitions
  - choose the largest partition
  - and halve this partition
- Update neighbor links
- Update left and right forward and backward edges
  - using the routing information of the neighbors
- Lemma
  - Insertion of a peer in Distance-Halving needs at most $O(\log^2 n)$ steps and messages.

Distance-Halving

- Simple and efficient P2P-Network
  - Degree O(1)
  - Diameter O(log n)
  - Load balancing
  - Lookup O(log n)
  - Insertion $O(\log^2 n)$
- Principle of continuous graph is also used in other peer-to-peer networks
  - Chord
  - Koorde
  - ViceRoy
- Here, formalized for the first time

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Principle of Multiple Choice

- For insertion every peer randomly chooses c log n intervals
- The peer chooses the largest interval and halves it
- Lemma
  - W.h.p. the interval size is at most a constant factor larger of smaller than the average size.

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Principle of Multiple Choice

- Lemma
  - After insertion of $n=2^k$ peers the interval sizes are 1/(2n), 1/n or 2/n whp.
- Proof
  - 1. part: Whp there is no interval larger than 2/n
  - Use the following lemma*
  - Lemma*
    - Let c/n be the largest interval. Then after insertion of 2n/c peers all intervals are at most c/(2n) whp.
  - Apply this lemma for c= n/2,n/4, ...,4

Principle of Multiple Choice

- Lemma
  - After insertion of $n=2^k$ peers the interval sizes are 1/(2n), 1/n or 2/n whp.
- Proof
  - 2. part: Whp. there is no smaller interval than 1/(2n) entstehen
  - Such an interval can be chosen with probability of at most 1/2
  - The probability that c log n intervals repeatedly hit such an interval is at most

$$2^{-c\log n} = n^{-c}$$

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