Peer-to-Peer Networks
15 Self-Organization

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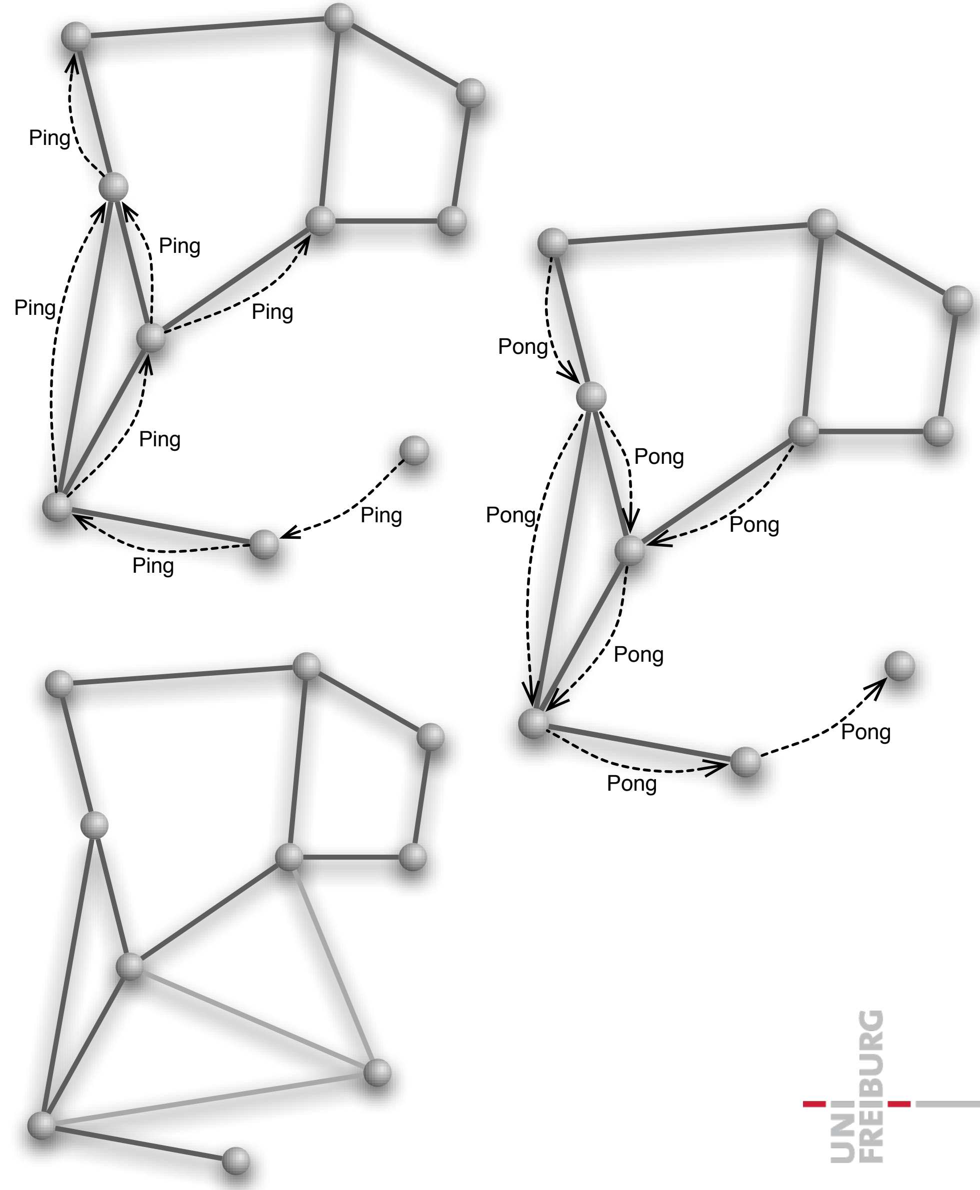
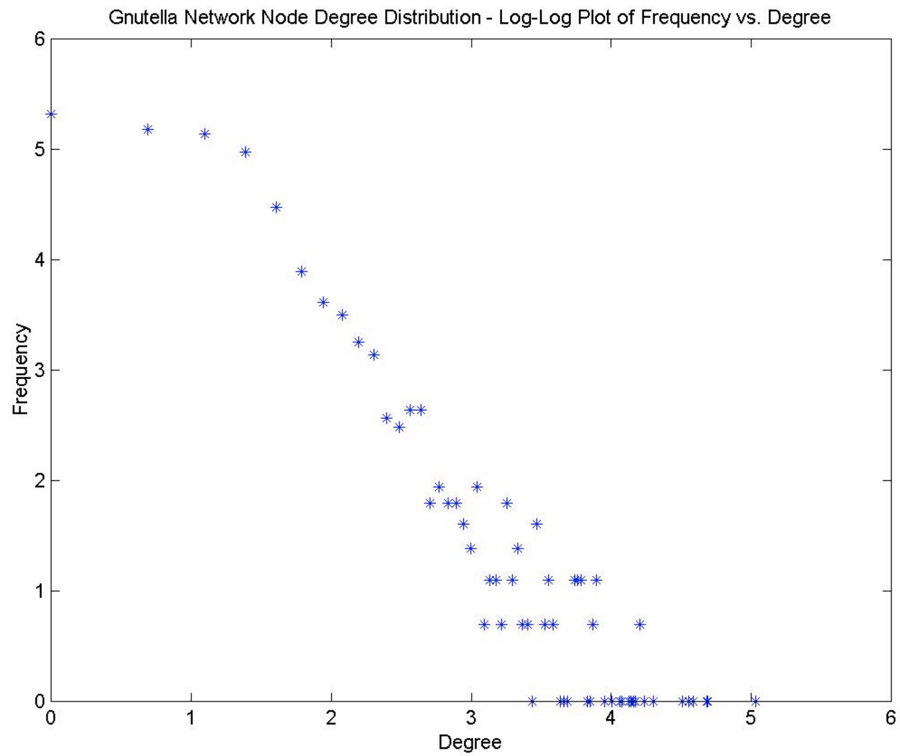
Gnutella — Connecting

- Protokoll
  - Ping
    - participants query for neighbors
    - are forwarded according for TTL steps (time to live)
  - Pong
    - answers Ping
    - is forwarded backward on the query path
    - reports IP and port adress (socket pair)
    - number and size of available files



Degree Distribution in Gnutella

- Modeling Large-scale Peer-to-Peer Networks and a Case Study of Gnutella
  - Mihajlo A. Jovanovic, Master Thesis, 2001
- The number of neighbors is distributed according a power law (Pareto) distribution
  - log(#peers with degree d) = c - k log d
  - #peers with degree d = C/d^k



Pareto-Distribution Examples

- Pareto 1897: Distribution of wealth in the population
- Yule 1944: frequency of words in texts
- Zipf 1949: size of towns
- length of molecule chains
- file length of Unix-system files
- ….

Pareto Verteilung

- Discreet Pareto-Distribution for $x \in \{1,2,3,\ldots\}$

$$\mathbf{P}[X = x] = \frac{1}{\zeta(\alpha) \cdot x^{\alpha}}$$

  - with constant factor

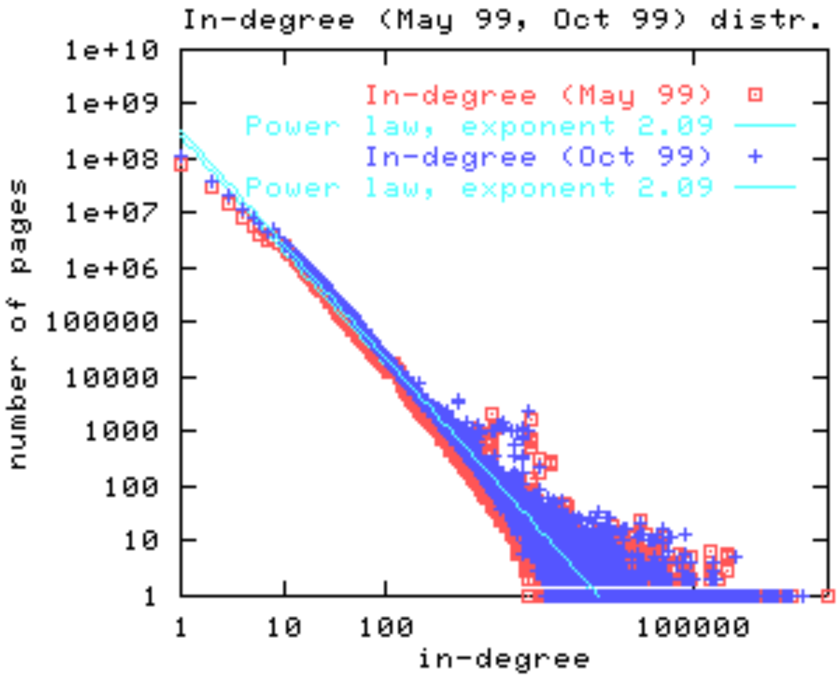
$$\zeta(\alpha) = \sum_{i=1}^{\infty} \frac{1}{i^{\alpha}}$$

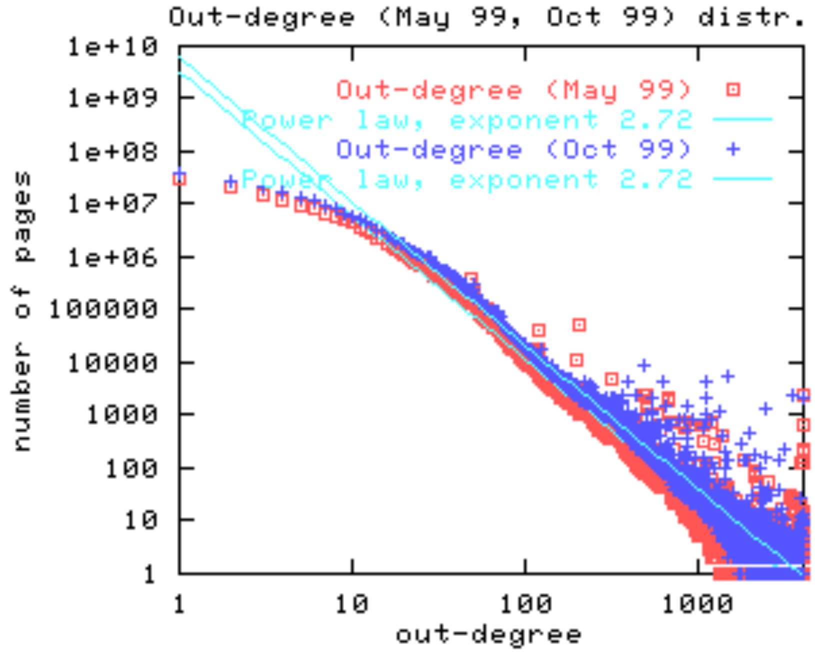
  - (also known as Riemann´s Zeta-function)
- Heavy tail property
  - not all moments $E[X^k]$ exist
  - the expectation exists if and only if (iff) $\alpha>2$
  - variance and $E[X^2]$ exist iff $\alpha>3$
  - $E[X^k]$ exists iff $\alpha>k+1$
- Density function of the continuous function for $x>x_0$

$$f(x) = \frac{\alpha - 1}{x_0} \left(\frac{x_0}{x}\right)^{\alpha}$$

Indegree and Outdegree of Web-Pages

- are described by a power law (Pareto) distribution





- Experiments of
  - Kumar et al 97: 40 millions Webpages
  - Barabasi et al 99: Domain *.nd.edu + Web-pages in distance 3
  - Broder et al 00: 204 millions web pages (Scan Mai und Okt. 1999)

Connectivity of Pareto Graphs

- William Aiello, Fan Chung, Linyuan Lu, A Random Graph Model for Massive Graphs, STOC 2000
- Undirected graph with n nodes where
  - the probability of k neighbors for a node is p_k
  - where $p_k = c\, k^{-\tau}$ for some normalizing factor c
- Theorem
  - For sufficient large n such Pareto-Graphs with exponent τ we observe
    - for $\tau < 1$ the graph is connected with probability 1-o(1)
    - for $\tau > 1$ the graph is nont connected with probability 1-o(1)
    - for $1< \tau <2$ there is a connected component of size $\Theta(n)$
    - for $2< \tau < 3.4785$ there is only one connected component of size $\Theta(n)$ and all others have size $O(\log n)$
    - for $\tau >3.4785$: there is no large connected component of size $\Theta(n)$ with probability 1-o(1)
    - For $\tau >4$: no large connected components which size can be described by a power law (Pareto) distribution

Zipf Distribution as a Variant of Power Laws

- George Kinsley Zipf claimed
  - that the frequency of the n most frequent word f(n)
  - satisfies the equation n f(n) = c.
- Zipf probability distribution for $x \in \{1,2,3,\ldots\}$

$$\mathbf{P}[X = x] = \frac{c}{x}$$

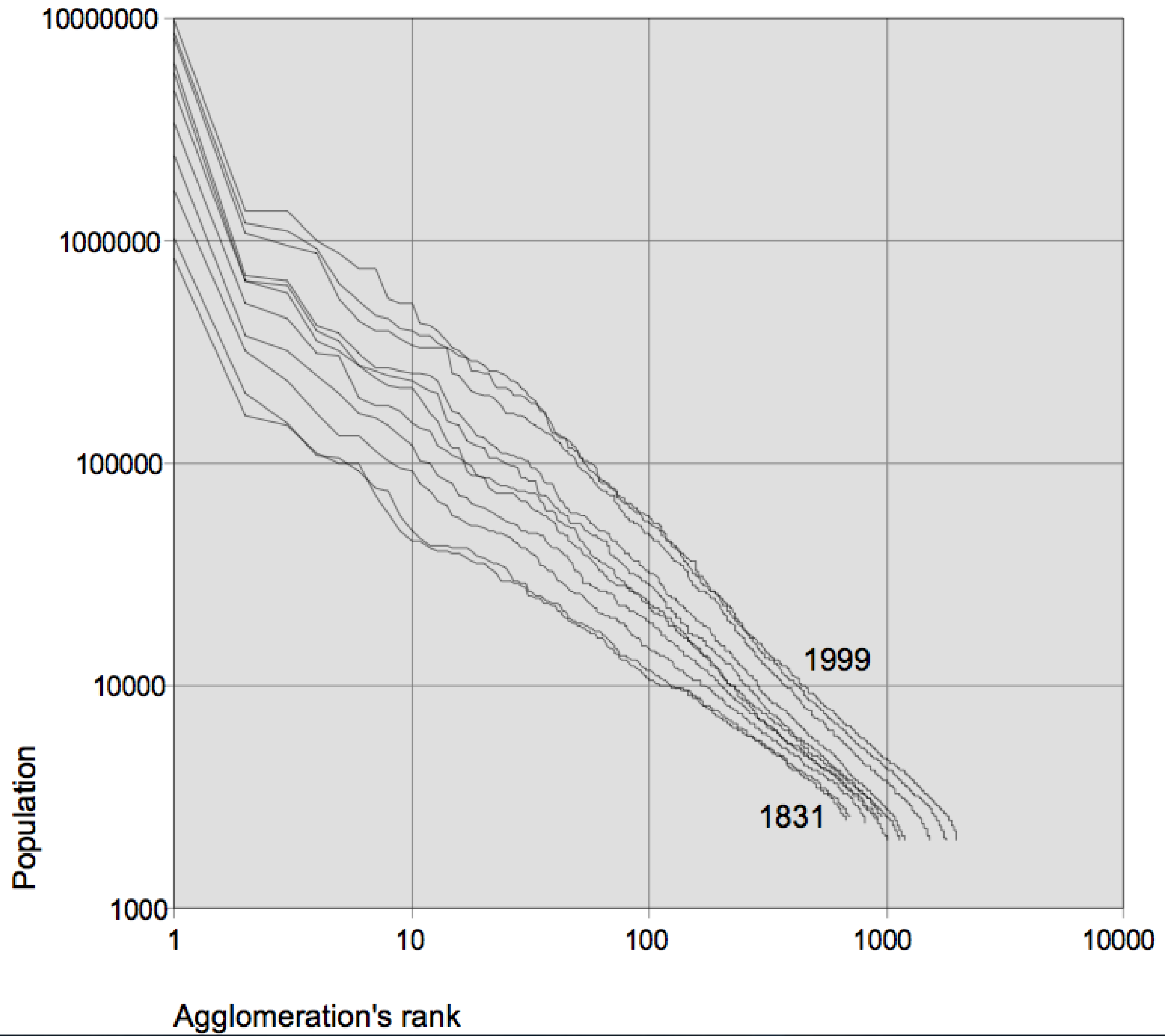
  - with constant factor c only defined for connstant sized sets, since
  - is unbounded

$$\ln n \leq \sum_{i=1}^{n} \frac{1}{i} \leq 1 + \ln n$$

- Zipf distribution relate to the rank
  - The Zipf exponent α may be larger than 1, i.e. $f(n) = c/n^{\alpha}$
- Pareto distribution realte the absolute size, e.g. the number of inhabitants

Size of towns
Scaling Laws and Urban Distributions, Denise Pumain, 2003

Figure 1 The hierarchical differentiation in urban systems:
Rank-size distribution of French agglomerations (1831-1999)



Zipf distribution

Zipf's Law and the Internet

Lada A. Adamic, Bernardo A. Huberman, 2002

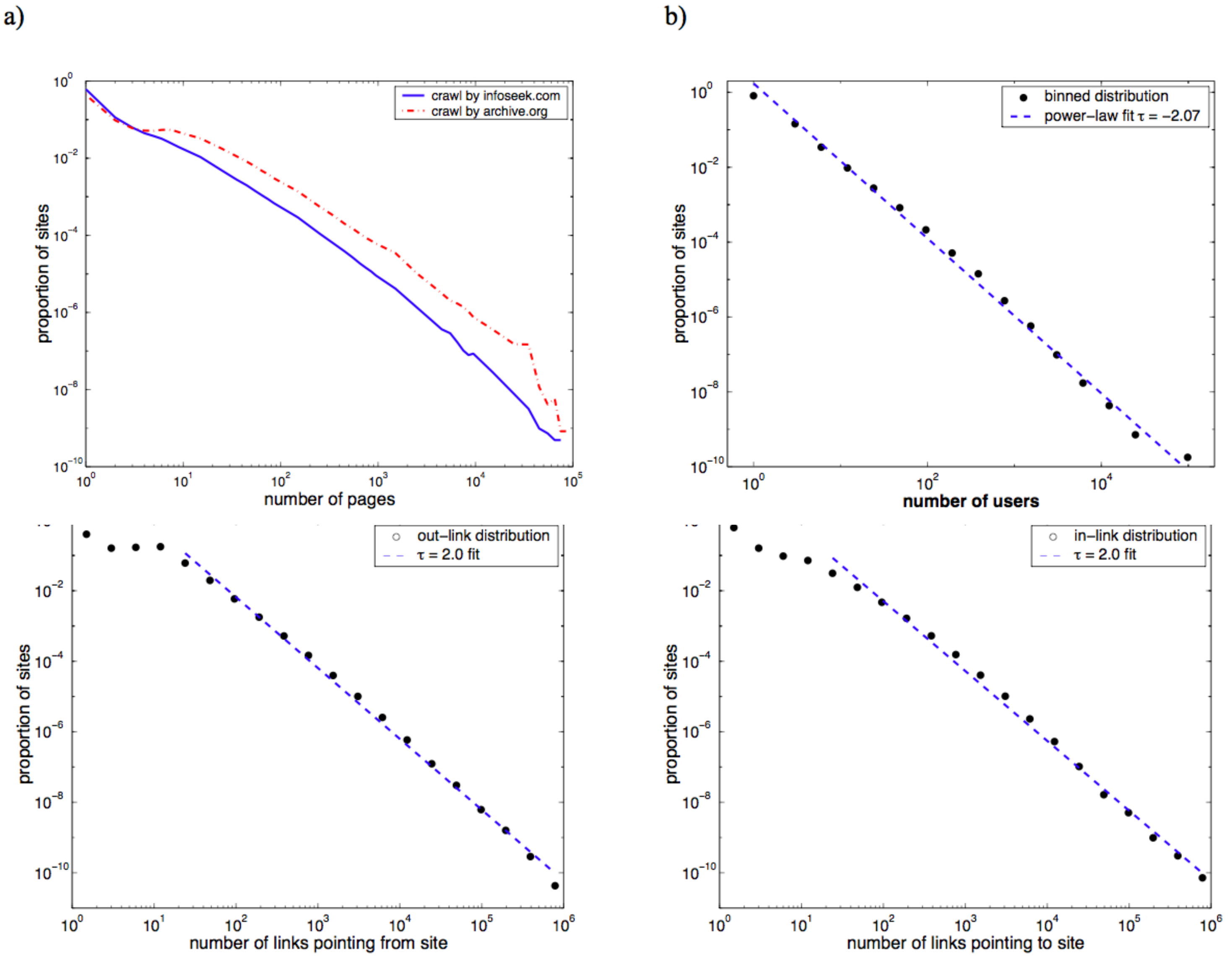
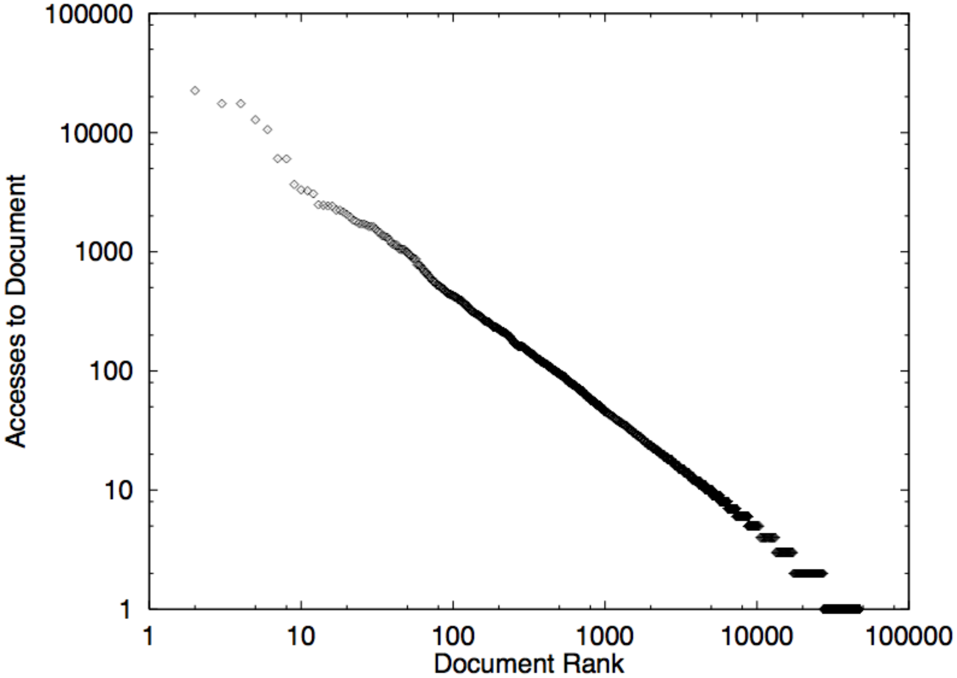
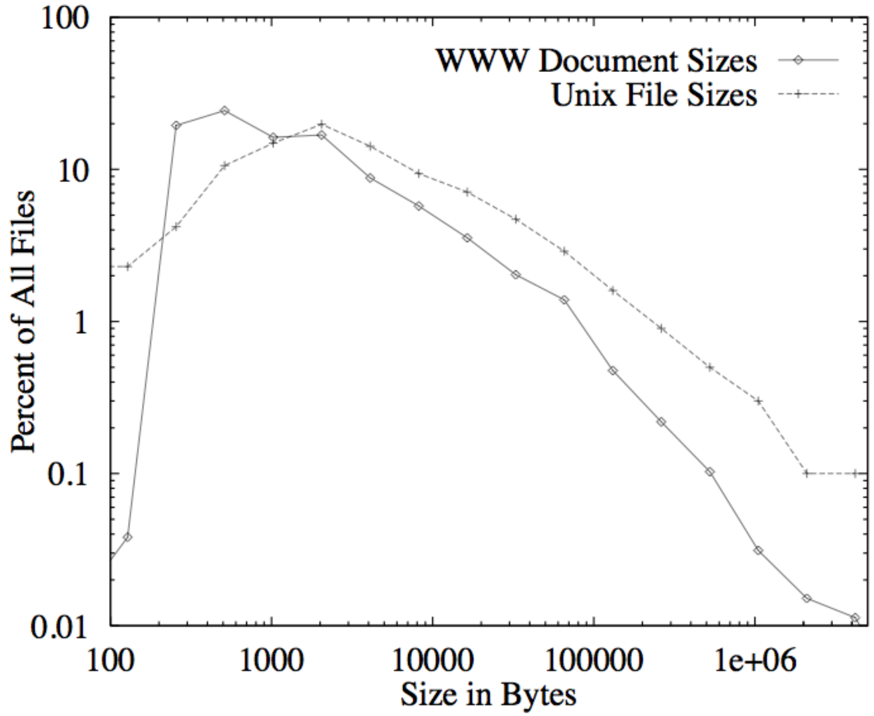


Figure 1. Fitted power law distributions of the number of site a) pages, b) visitors, c) out links, and d) in links, measured in 1997.

Pareto Distribution!!

Heavy-Tailed Probability Distributions in the World Wide Web
Mark Crovella, Murad, Taqqu, Azer Bestavros, 1996





Small World Phenomenon

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- Milgram's experiment 1967
  - 60 random chosen participants in Wichita, Kansas had to send a packet to an unknown address
  - They were only allowed to send the packet to friends
    - likewise the friends
- The majority of packtes arrived within six hops
- Small-World-Networks
  - are networks with Pareto distributed node degree
  - with small diameter (i.e. $O(\log^c n)$)
  - and relatively many cliques
- Small-World-Networks
  - Internet, World-Wide-Web, nervous systems, social networks

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How do Small World Networks Come into Existence?

- Emergence of scaling in random networks, Albert-Laszlo Barabasi, Reka Albert, 1999
- Preferential Attachment-Modell (Barabasi-Albert):
  - Starting from a small starting graph successively nodes are inserte with m edges each (m is a parameter)
  - The probability to choose an existing node as a neighbor is proportional to the current degree of a node
- This leads to a Pareto network with exponent 2,9 ± 0,1
  - however cliques are very seldom
- Watts-Strogatz (1998)
  - Start with a ring and connections to the m nearest neighbors
  - With probability p every edge is replaced with a random edge
  - Allows continuous transition from an ordered graph to chaos
- Extended by Kleinberg (1999) for the theoretical verification of Milgram‘s experiment

Analyzing Gnutella

- Modeling Large-scale Peer-to-Peer Networks and a Case Study of Gnutella
  - Mihajlo A. Jovanovic, 2001

Snapshot date	Nodes	Edges	Diameter
11/13/2000	992	2465	9
11/16/2000	1008	1782	12
12/20/2000	1077	4094	10
12/27/2000	1026	3752	8
12/28/2000	1125	4080	8

Analyzing Gnutella

- Modeling Large-scale Peer-to-Peer Networks and a Case Study of Gnutella
  - Mihajlo A. Jovanovic, 2001
- Comparison of the characteristic path length
  - mean distance between two nodes

	Gnutella	BA	WS	G(n,p)	2D mesh
11/13/2000	3.72299	3.47491	4.59706	4.48727	20.6667
11/16/2000	4.42593	4.07535	4.61155	5.5372	21.3333
12/20/2000	3.3065	3.19022	4.22492	3.6649	22
12/27/2000	3.30361	3.18046	4.19174	3.70995	21.3333
12/28/2000	3.32817	3.20749	4.25202	3.7688	22.6667

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