Wireless Sensor Networks
1. Basics

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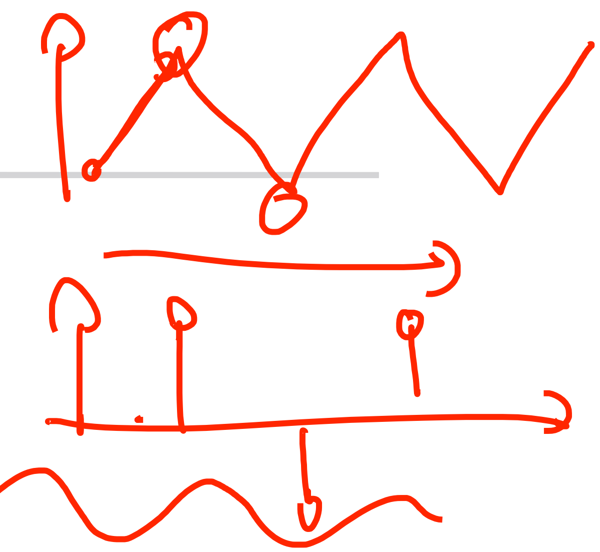
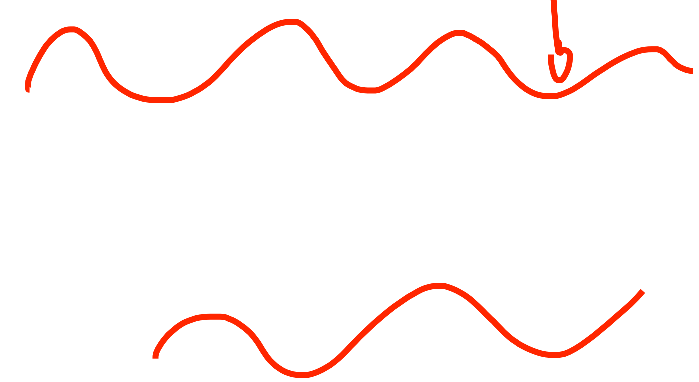
Version 17.04.2016

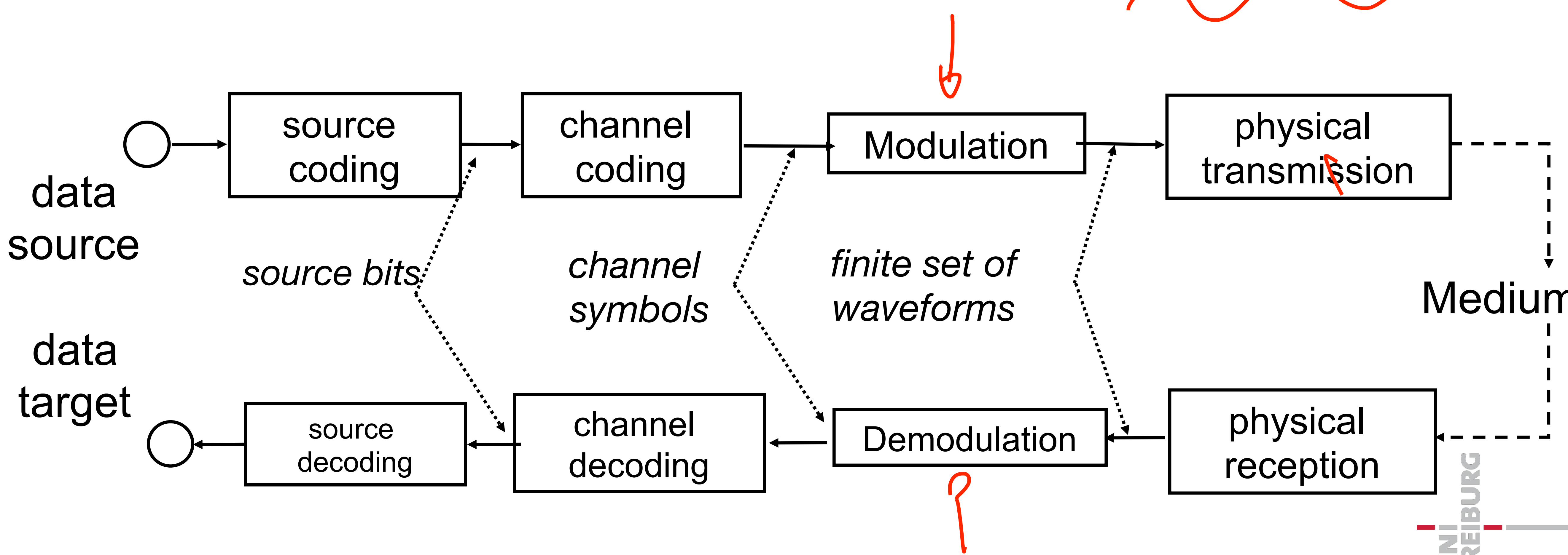
Structure of a *Broadband* Digital transmission

Baseband

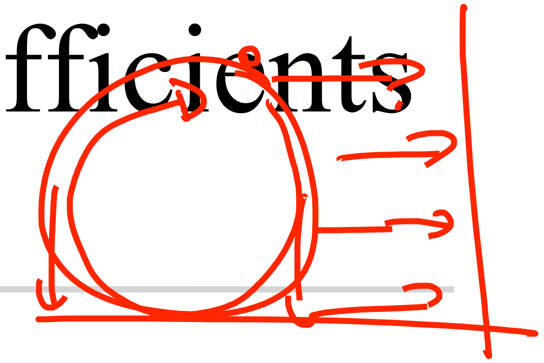


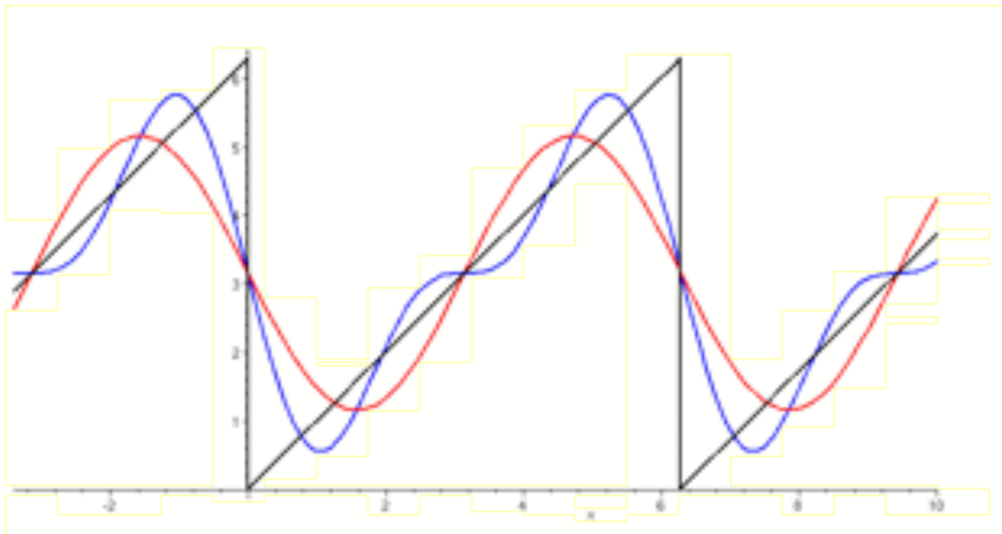


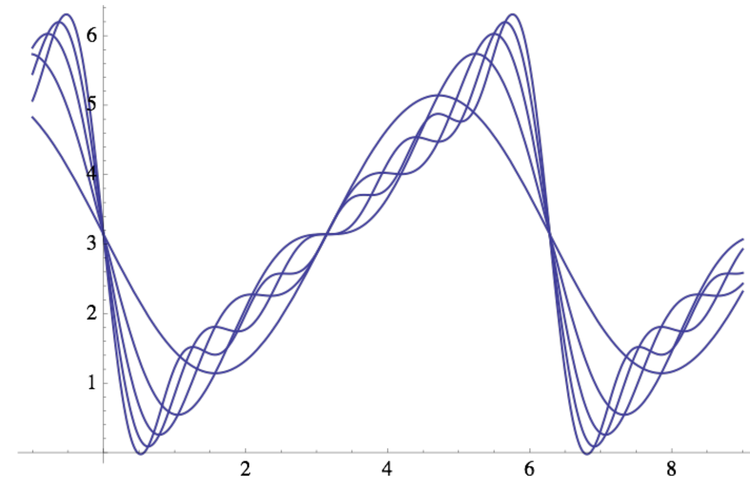
- MOdulation/DEModulation
  - Translation of the channel symbols by
    - amplitude modulation
    - phase modulation
    - frequency modulation
    - or a combination thereof



Computation of Fourier Coefficients

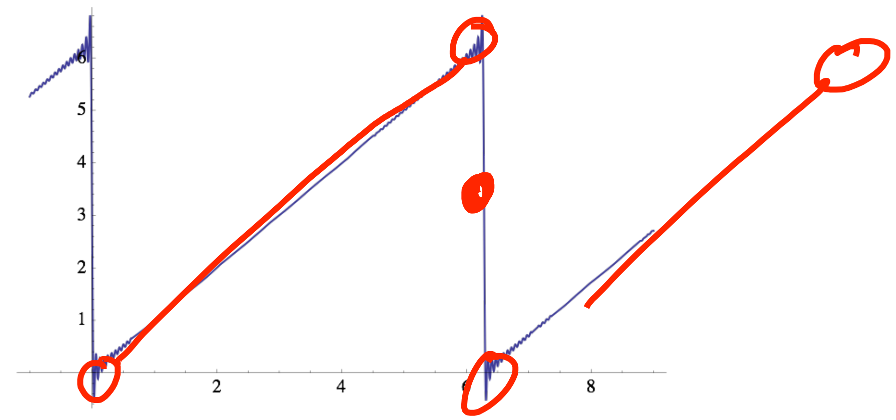






$f(x) = x$, für $0 < x < 2\pi$

$$f(x) = \pi - 2\left(\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \dots\right)$$



$$\frac{d \sin x}{dx} = \cos x$$

$$\frac{d \cos x}{dx} = -\sin x \to -\cos x \to \sin x$$

Fourier Analysis for General Period

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- Theorem of Fourier for period T=1/f:
  - The coefficients c, a_n, b_n are then obtained as follows

$$g(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(2\pi k f t) + b_k \sin(2\pi k f t)$$

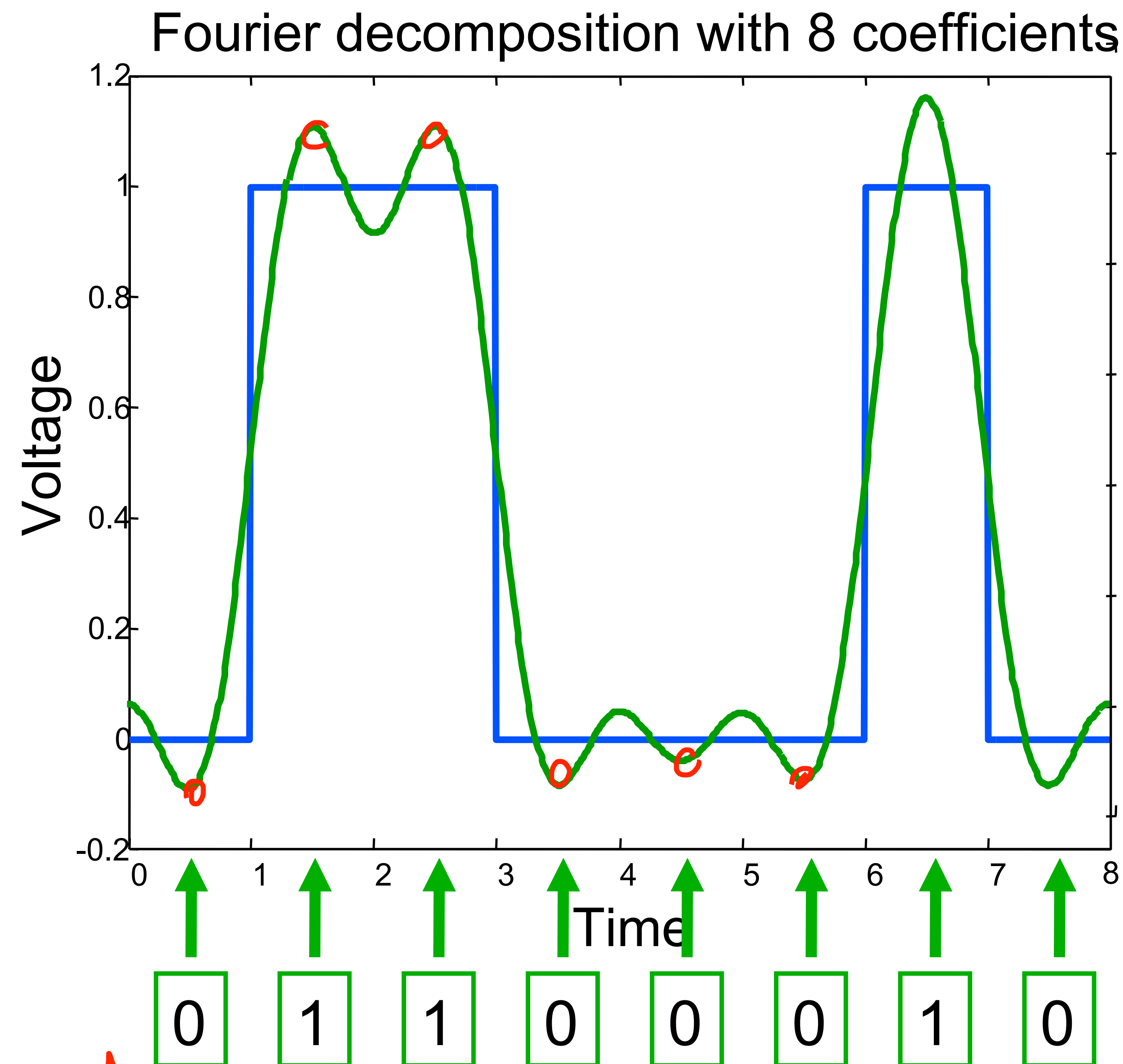
$$a_k = \frac{2}{T} \int_0^T g(t) \cos(2\pi n f t) dt$$

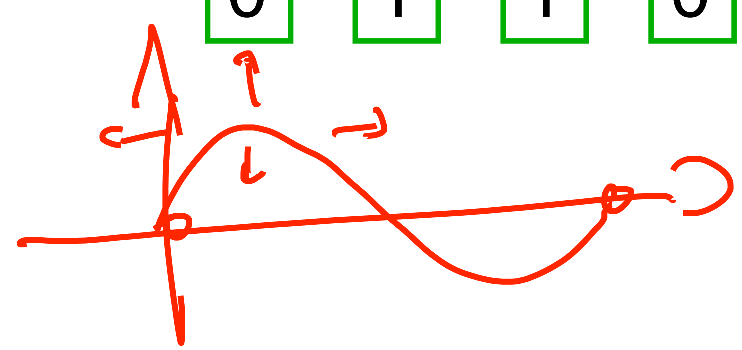
$$b_k = \frac{2}{T} \int_0^T g(t) \sin(2\pi n f t) dt$$

- The sum of squares of the k-th terms is proportional to the energy consumed in this frequency: $(a_k)^2 + (b_k)^2$

How often do you measure?

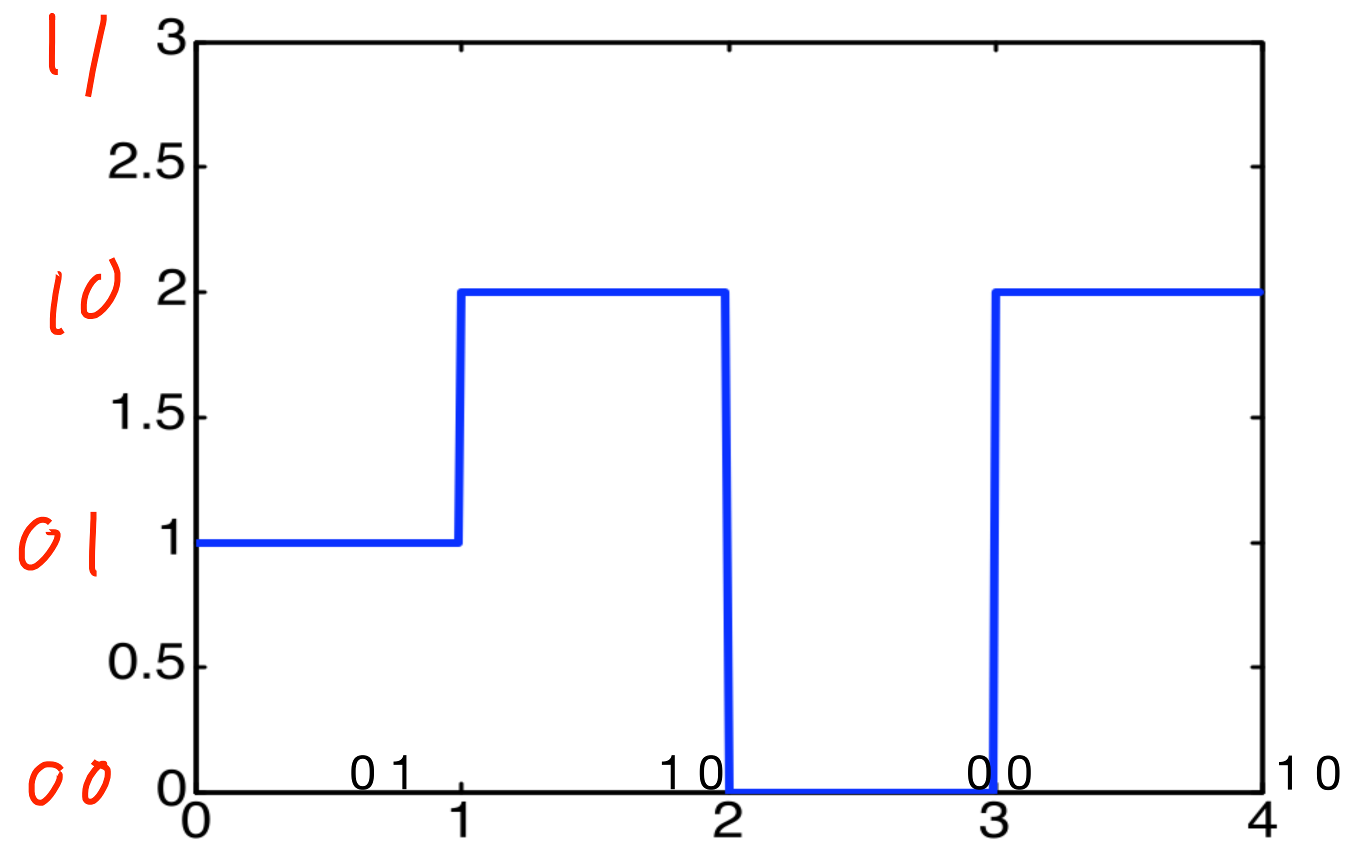
- How many measurements are necessary
  - to determine a Fourier transform to the k-th component, exactly?
- Nyquist-Shannon sampling theorem
  - To reconstruct a continuous band-limited signal with a maximum frequency f_{max} you need at least a sampling frequency of 2 f_{max}.





Symbols and Bits

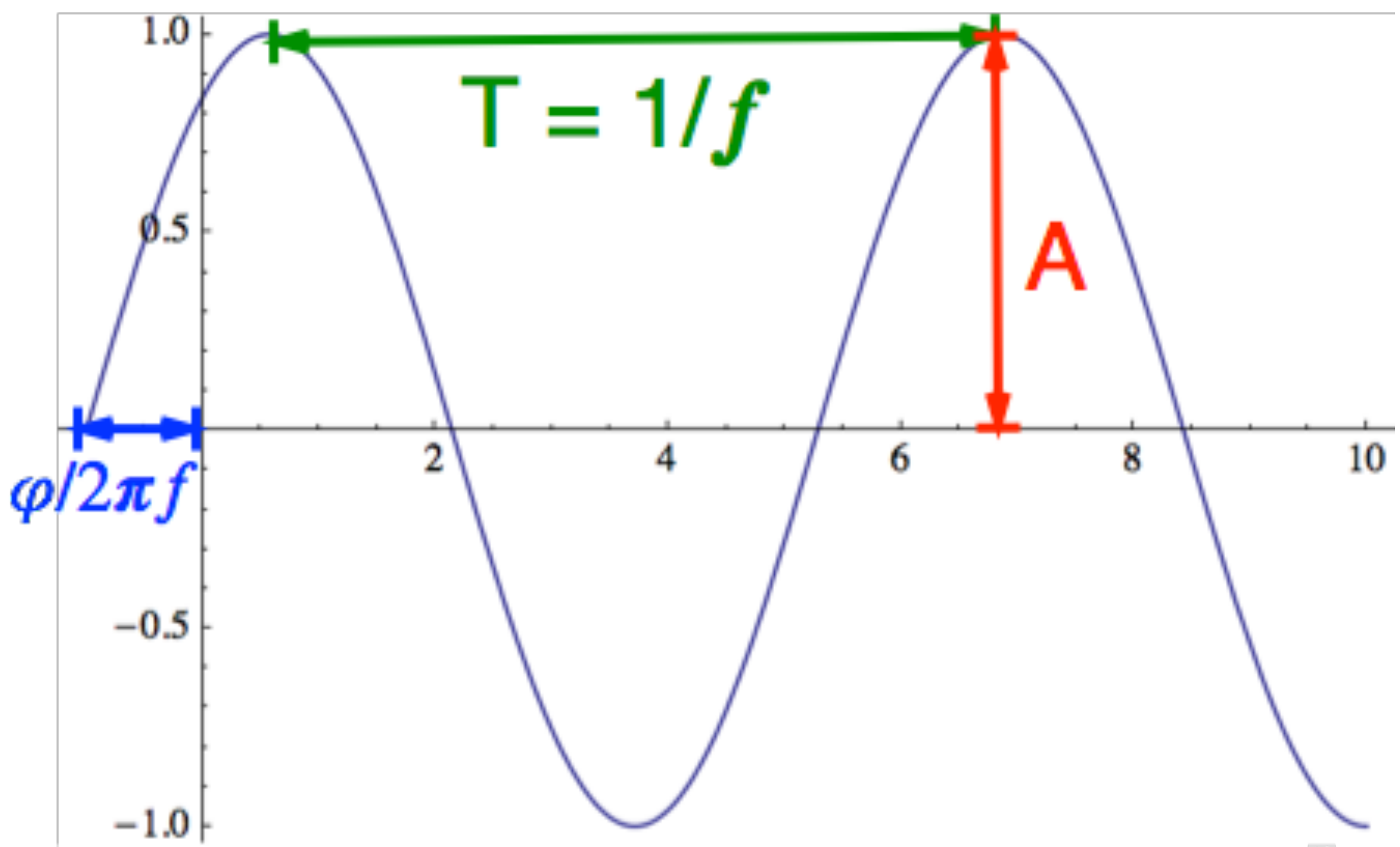
- For data transmission instead of bits can also be used symbols
  - E.g. 4 Symbols: A, B, C, D with
    - A = 00, B = 01, C = 10, D = 11
- Symbols
  - Measured in baud
  - Number of symbols per second
- Data rate
  - Measured in bits per second (bit / s)
  - Number of bits per second
- Example
  - 2400 bit/s modem is 600 baud (uses 16 symbols)



Broadband

- Idea
  - Focusing on the ideal frequency of the medium
  - Using a sine wave as the carrier wave signals
- A sine wave has no information
  - the sine curve continuously (modulated) changes for data transmission,
  - implies spectral widening (more frequencies in the Fourier analysis)
- The following parameters can be changed:
  - Amplitude A
  - Frequency f=1/T
  - Phase φ

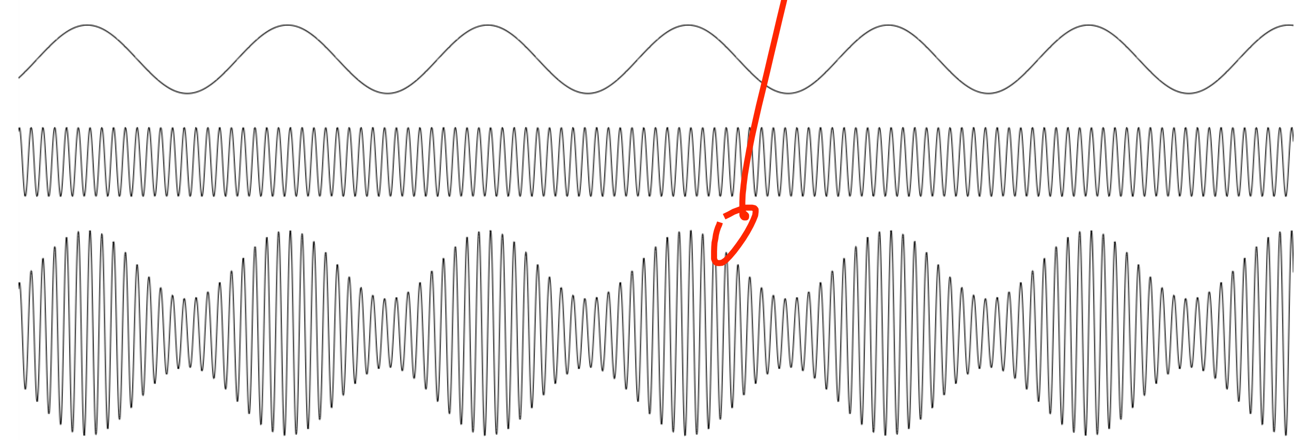
$$s(t) = A \sin(2\pi f t + \phi)$$



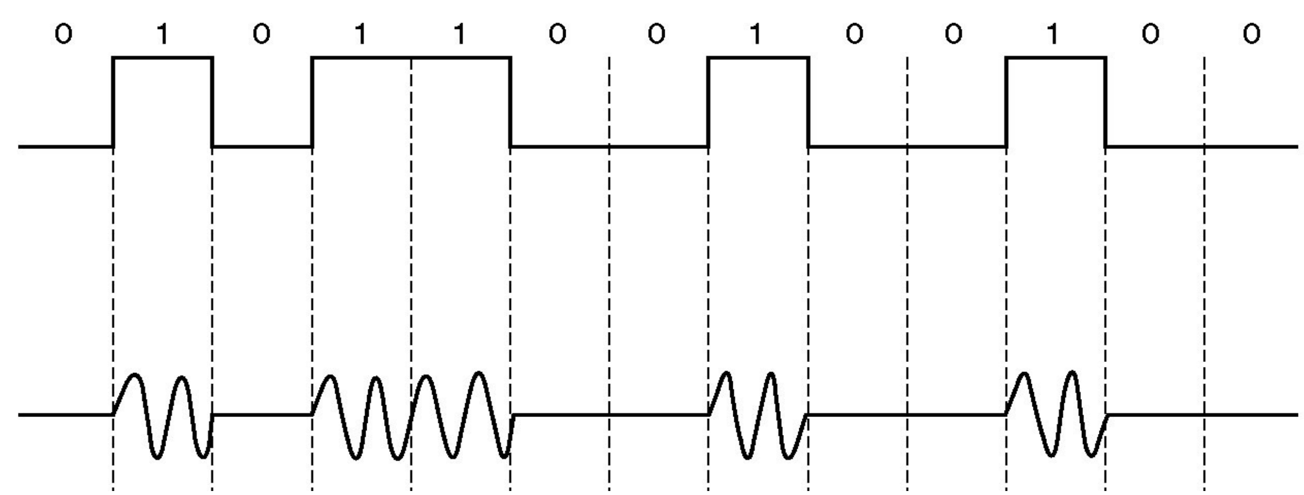
Amplitude Modulation

- The time-varying signal s (t) is encoded as the amplitude of a sine curve:

$$f_A(t) = s(t)\,\mathsf{sin}(2\pi f t + \phi)$$



- Analog Signal
- Digital signal
  - amplitude keying
  - special case: symbols 0 or 1
    - on / off keying



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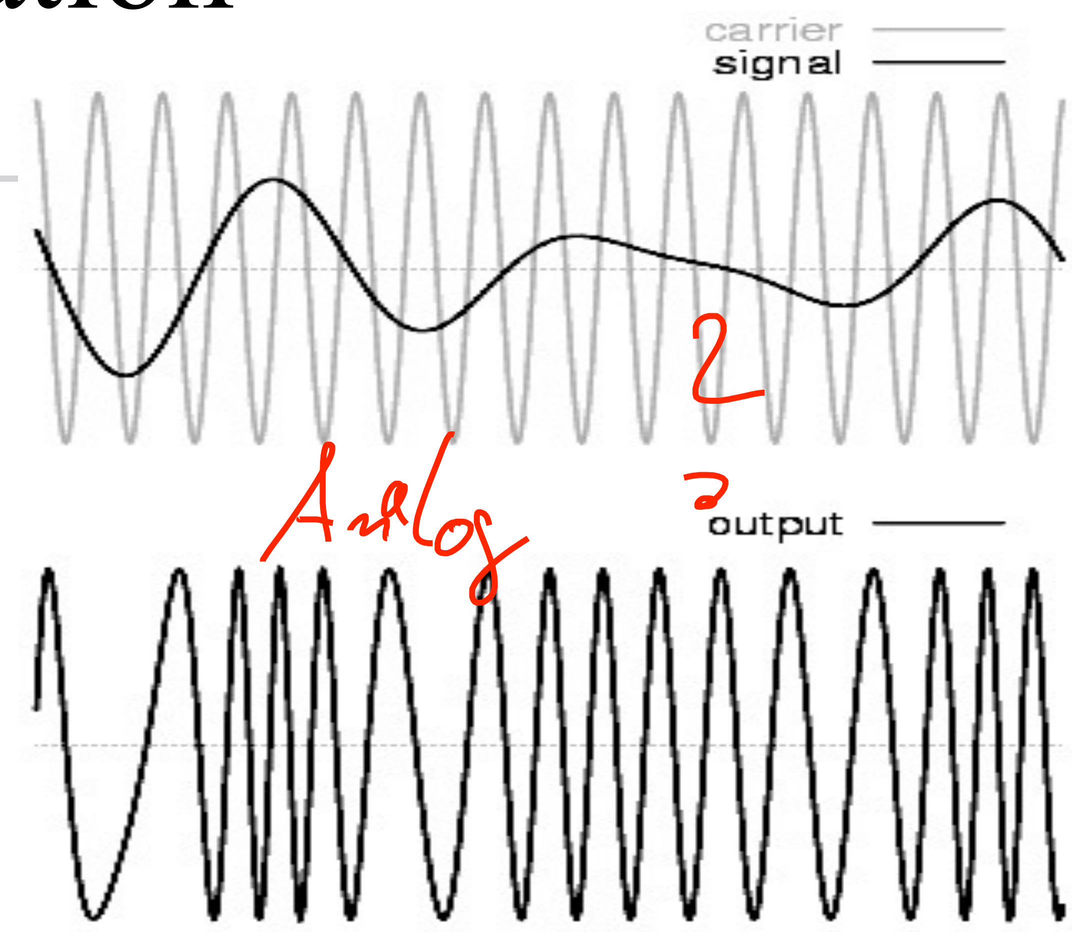
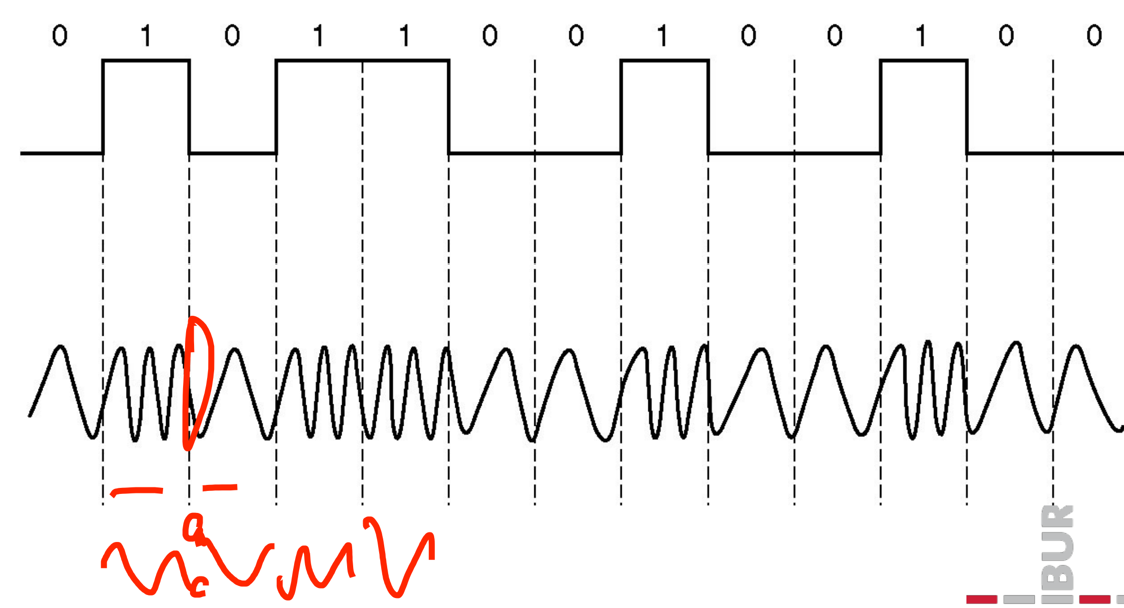
Frequency Modulation

- The time-varying signal s (t) is encoded in the frequency of the sine curve:

$$f_F(t) = a\,\sin(2\pi s(t)t + \phi)$$

- Analog signal
  - Frequency modulation (FM)
  - Continuous function in time
- Digital signal
  - Frequency Shift Keying (FSK)
  - E.g. frequencies as given by symbols





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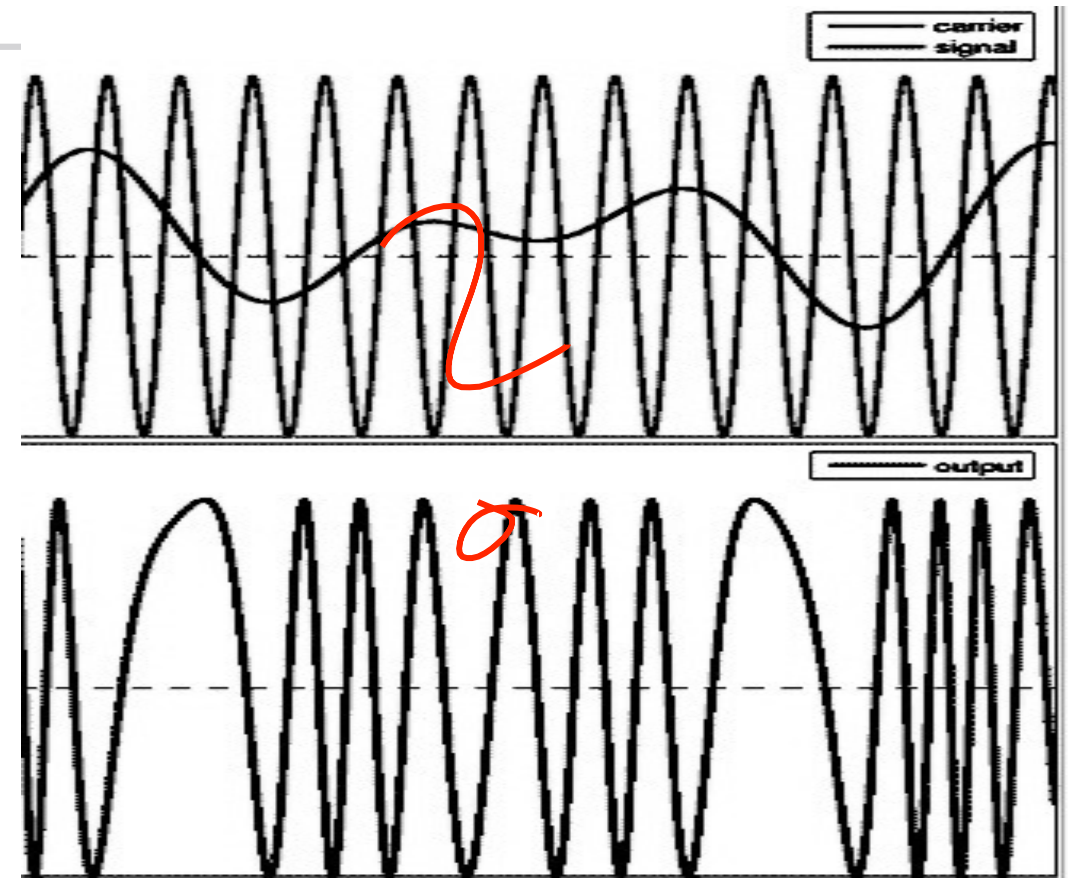
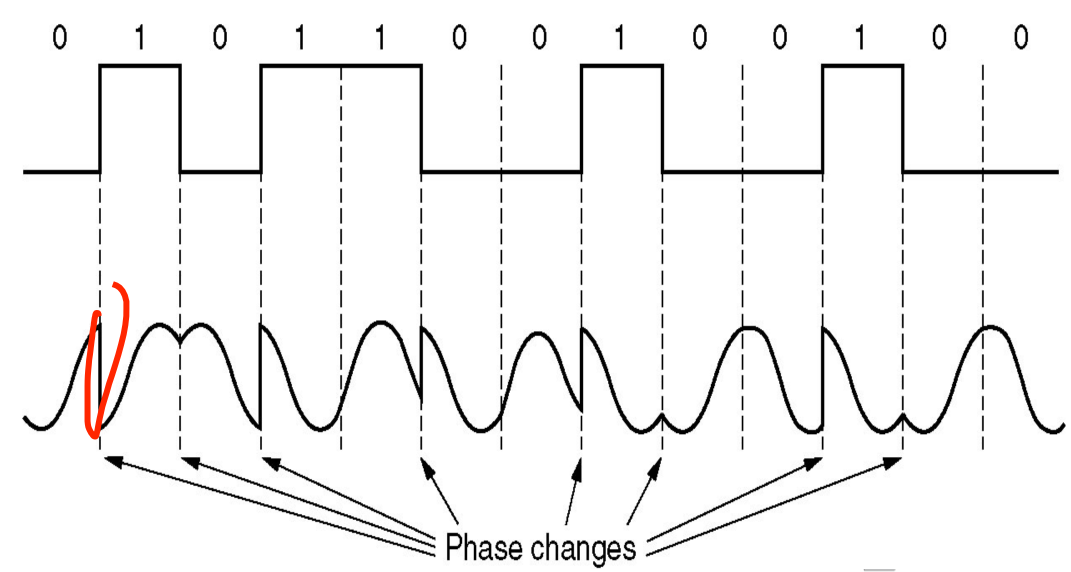
Phase Modulation

- The time-varying signal s (t) is encoded in the phase of the sine curve:

$$f_P(t) = a\,\mathsf{sin}(2\pi f t + s(t))$$

- Analog signal
  - phase modulation (PM)
  - very unfavorable properties
  - es not used
- Digital signal
  - phase-shift keying (PSK)
  - e.g. given by symbols as phases

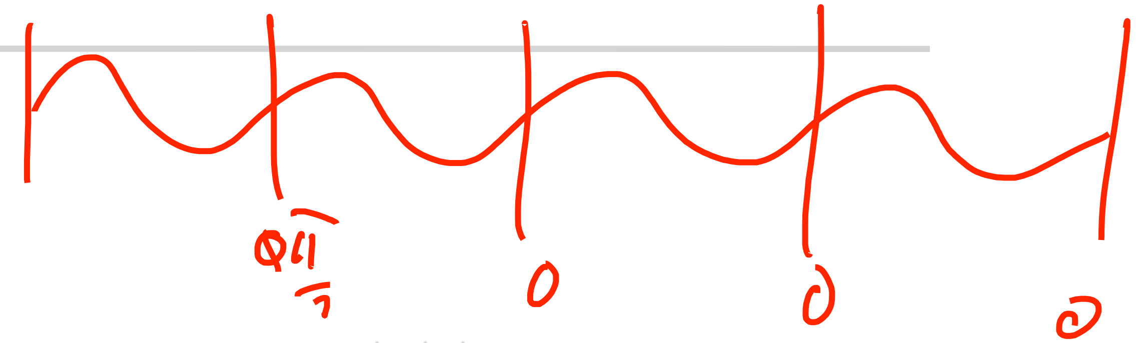




Digital and Analog signals in Comparison

- For a station there are two options
  - digital transmission
    - finite set of discrete signals
    - e.g. finite amount of voltage sizes / voltages
  - analog transmission
    - Infinite (continuous) set of signals
    - E.g. Current or voltage signal corresponding to the wire
- Advantage of digital signals:
  - There is the possibility of receiving inaccuracies to repair and reconstruct the original signal
  - Any errors that occur in the analog transmission may increase further

Phase Shift Keying (PSK)

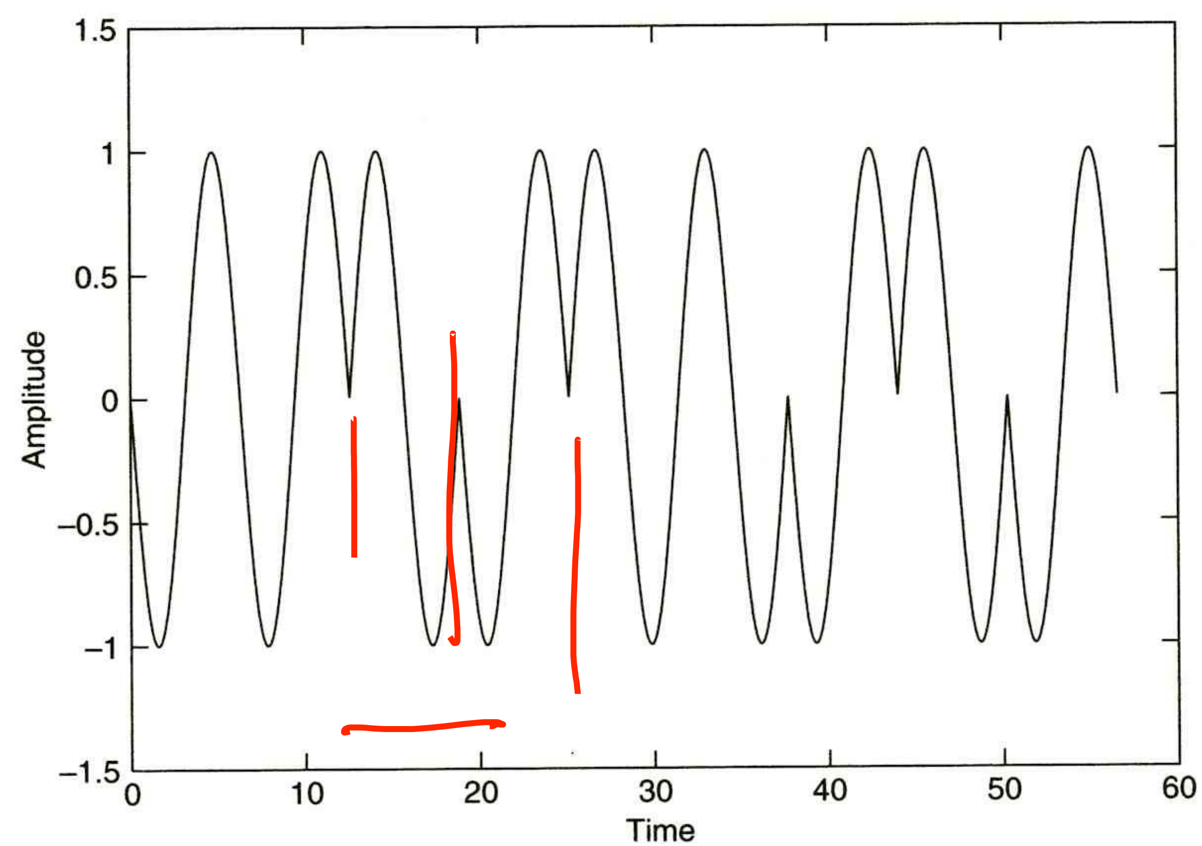


- For phase signals $\phi_i(t)$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cdot \sin(\omega_0 t + \phi_i(t))$$

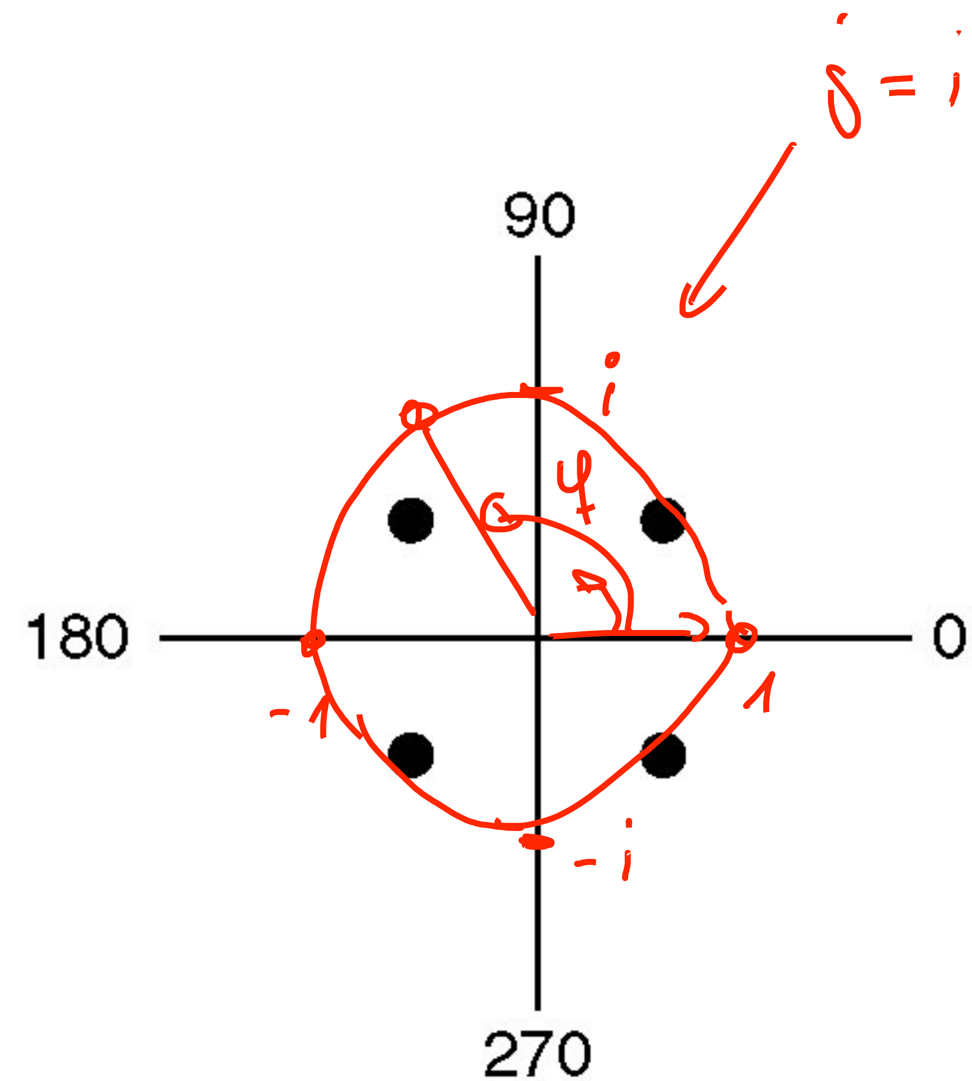
→ Synchronization

- Example:



PSK with Different Symbols

- Phase shifts can be detected by the receiver very well
- Encoding various Symoble very simple
  - Using phase shift e.g. π / 4, 3/4π, 5/4π, 7/4π
    - rarely: phase shift 0 (because of synchronization)
  - For four symbols, the data rate is twice as large as the symbol rate
- This method is called Quadrature Phase Shift Keying (QPSK)



Amplitude and Phase Modulation

- Amplitude and phase modulation can be successfully combined
  - Example: 16-QAM (Quadrature Amplitude Modulation)
    - uses 16 different combinations of phases and amplitudes for each symbol
    - Each symbol encodes four bits ($2^4 = 16$)
  - The data rate is four times as large as the symbol rate

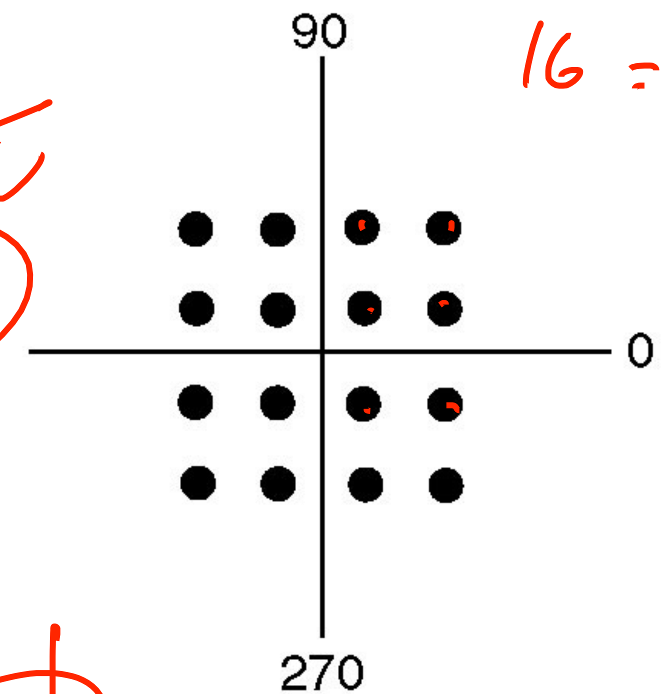
Energy of nois

256-QAM

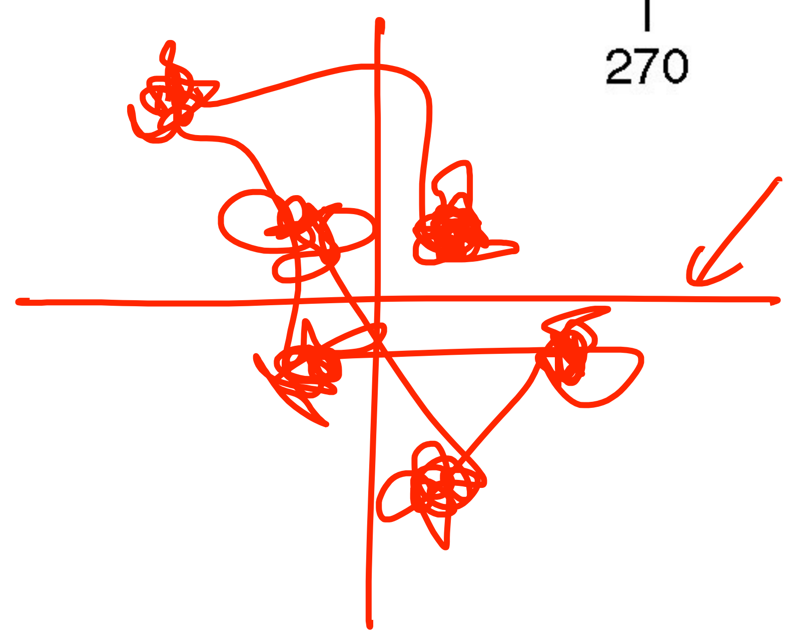
$16 = 2^4$

$\times \log_2 t$

$\frac{S}{N}$



noise



Nyquist‘s Theorem

- Definition
  - The band width H is the maximum frequency in the Fourier decomposition
- Assume
  - The maximum frequency of the received signal is f = H in the Fourier transform
    - (Complete absorption [infinite attenuation] all higher frequencies)
  - The number of different symbols used is V
  - No other interference, distortion or attenuation of
- Nyquist theorem
  - The maximum symbol rate is at most 2 H baud.
  - The maximum possible data rate is a bit more than $2 \log_2 H V$ / s.

Do more symbols help?

- Nyquist's theorem states that could theoretically be increased data rate with the number of symbols used
- Discussion:
  - Nyquist's theorem provides a theoretical upper bound and no method of transmission
  - In practice there are limitations in the accuracy
  - Nyquist's theorem does not consider the problem of noise

The Theorem of Shannon

- Indeed, the influence of the noise is fundamental
  - Consider the relationship between transmission intensity S to the strength of the noise N
  - The less noise the more signals can be better recognized
- Theorem of Shannon
  - The maximum possible data rate is H $\log_2(1 + S / N)$ bits/s
    - with bandwidth H
    - Signal strength S
- Attention
  - This is a theoretical upper bound
  - Existing codes do not reach this value

$$\boxed{H \log_2\left(1 + \frac{S}{N}\right)}$$

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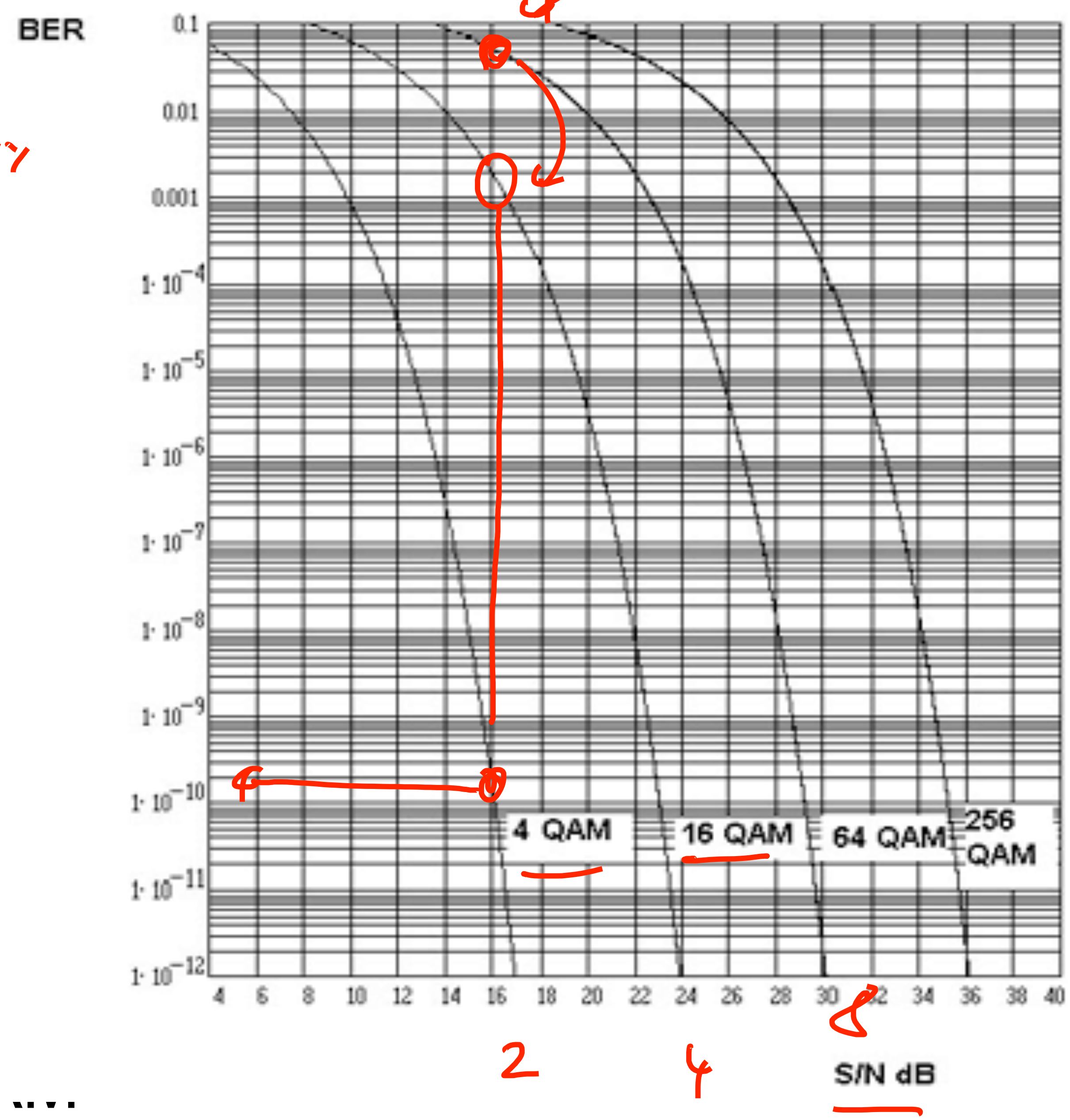
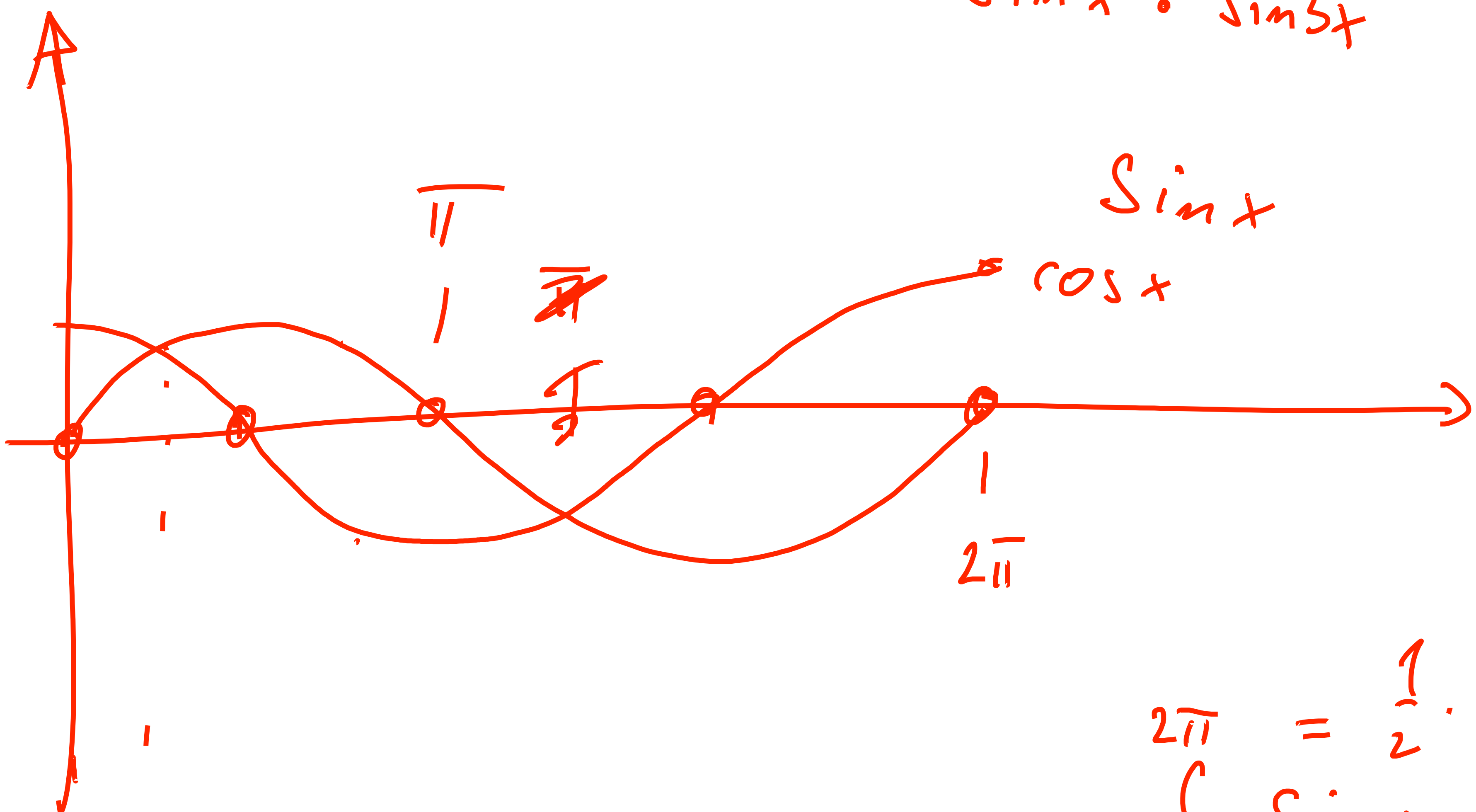
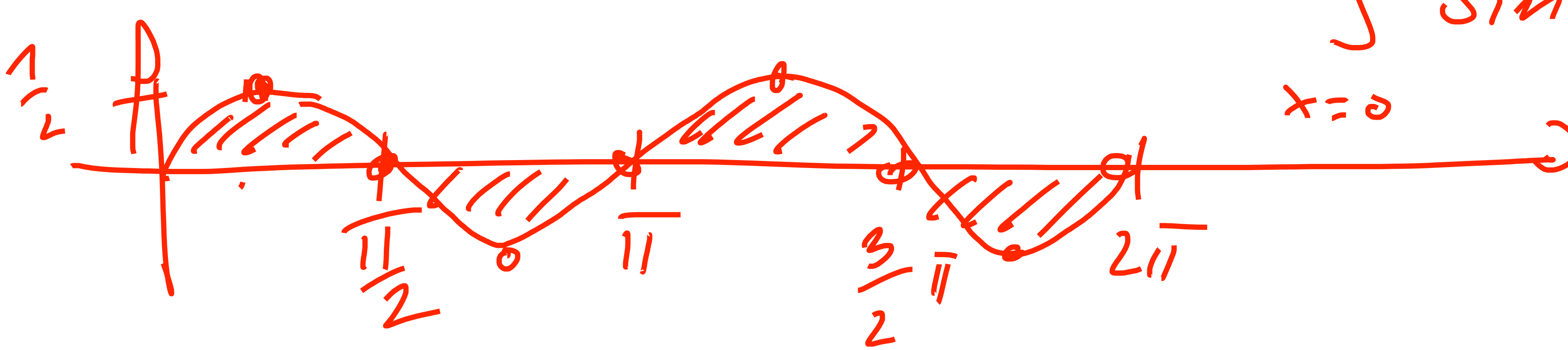
Bit Error Rate and SINR

- Higher SIR decreases Bit Error Rate (BER)
  - BER is the rate of faulty received bits
- Depends from the
  - signal strength
  - noise
  - bandwidth
  - encoding
- Relationship of BER and SINR
  - Example: 4 QAM, 16 QAM, 64 QAM, 256 Q. ….



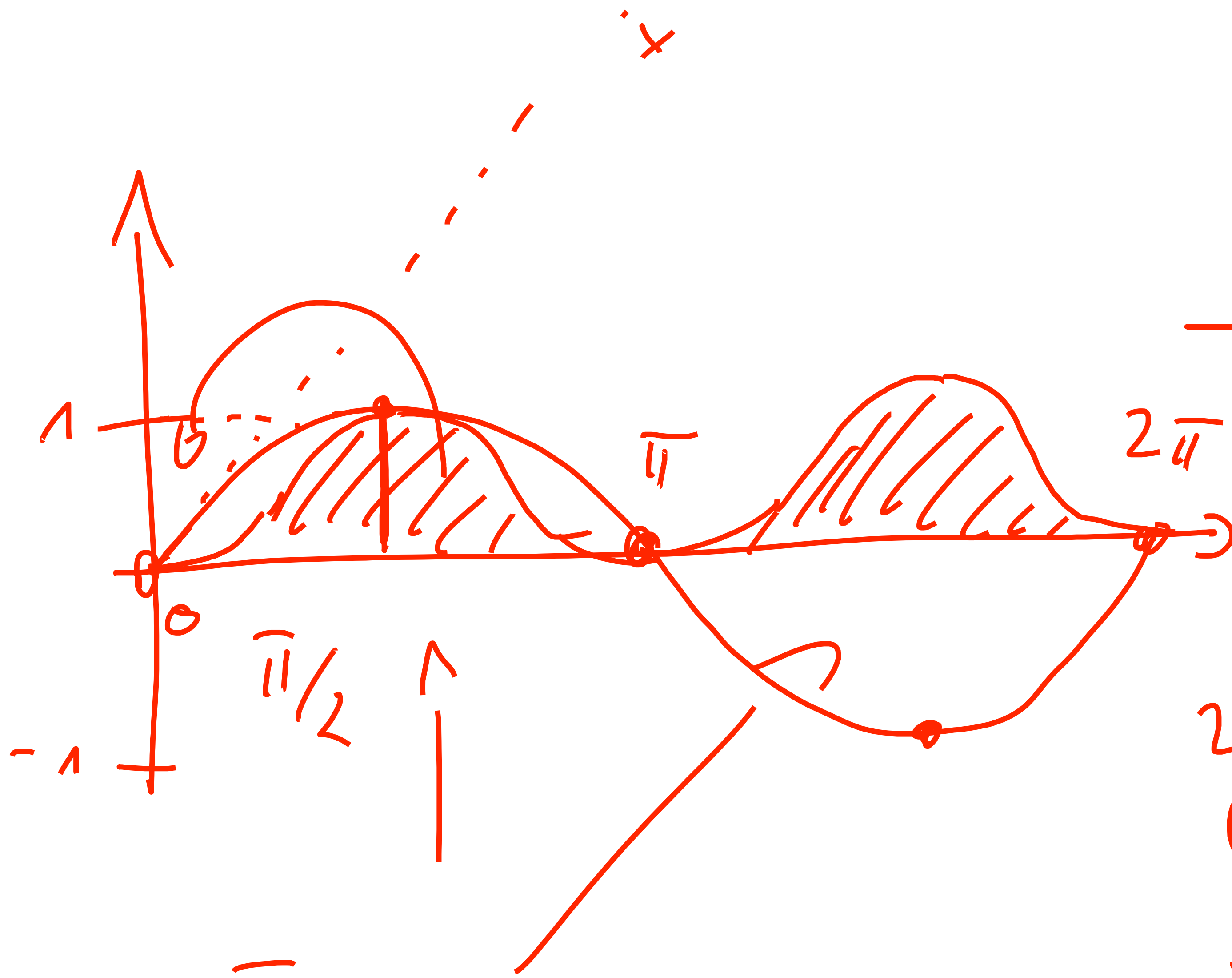
$\sin x \cdot \sin 3x \qquad 0 \cdot 1 = 0$





$$\int_{x=0}^{2\pi} \sin x \cdot \cos x \, dx \qquad = \frac{1}{2} \cdot \sin 2x$$

$= 0$

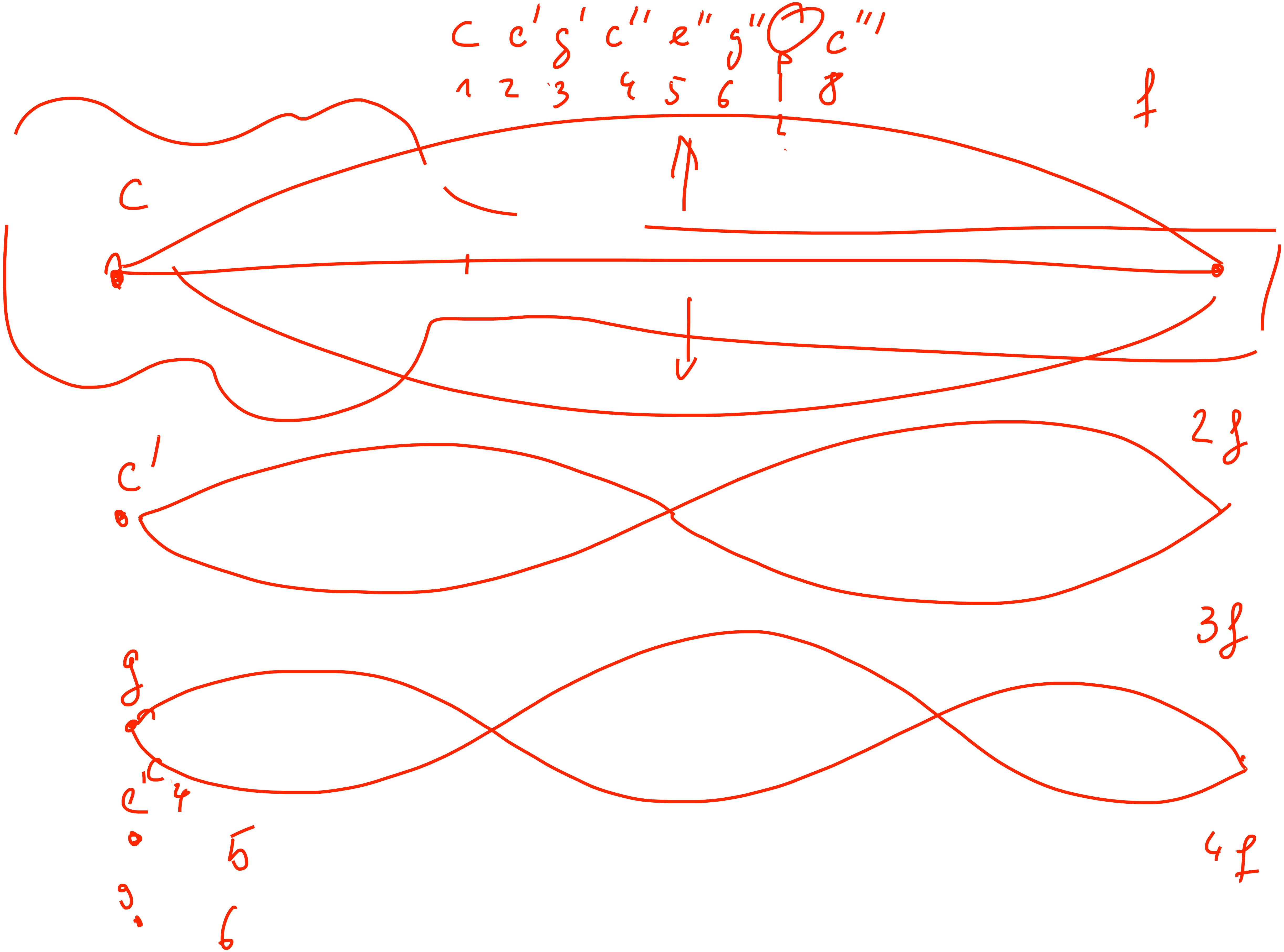


$\frac{\pi}{2} \cdot 1$

$(\sin x)(\sin x)$

$= 1 - \frac{1}{2} \cdot \cos(2x)$

$$\int_{x=1}^{2\pi} \sin x = \pi$$



c c' g' c'' e'' g'' c'''
1 2 3 4 5 6 7 8
f
c
2f
c'
3f
g
4f
4
5
6