Wireless Sensor Networks

2. Multiplexing

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Multiple Use of the Medium

- **Spatial Multiplexing**
  - Parallel and exclusive use of transmission channels
    - e.g. Extra lines / cells / directional antenna

- **Frequency division multiplexing**
  - Multiple signals to be transmitted in a frequency range of bundled;
  - In radio transmission, different frequencies are assigned to different stations.

- **Time division multiplexing**
  - Delayed transmission of multiple signals

- **Code division multiplexing**
  - Coding of the signal into orthogonal codes, which can now be broadcast simultaneously on one frequency
  - Decoding with overlay also possible

- **Multiple-Input Multiple-Output**
  - Sending and receiving antennas by several
  - Using the spatial and temporal information about location of several waves
    - e.g. 802.11n
Space

- Spatial distribution (space multiplexing)
  - Utilization of distance loss for the parallel operation of different radio cells → cellular networks
  - Using directional antennas for communications directed requested
  - GSM antennas with directional characteristics
  - Radio with a satellite dish
  - Laser communications
  - Infrared communication
Frequency Multiplexing

- Allocation of bandwidth in frequency sections
- Spread of the channels and hopping
  - Direct Sequence Spread Spectrum (DSSS)
  - Xor a signal with a pseudo-random number sequence at the transmitter and receiver (Relates to code-division multiplexing)
  - Other signals appear as background noise
- Frequency Hopping Spread Spectrum (FHSS)
  - Frequency change by pseudo-random numbers
  - two versions
    - Quick change (almost hopping): Multiple frequencies per user data bits
    - Slowly changing (slow hopping): Multiple user bits per frequency
Time Multiplexing

- Temporal distribution of sender-/receiver channel
- Participants receive exclusive periods (slots) on the media
- Accurate synchronization necessary
- Coordination necessary, or rigid division
Direct Sequence Spread Spectrum

- A chip is a bit sequence (given by \{-1, +1\}), which encode a smaller set of symbols
- E.g. Transmission signal: 0 = (+1,+1,-1), 1=(-1,-1,+1)
  
  \[
  \begin{array}{cccc}
  0 & 1 & 0 & 1 \\
  +1 & +1 & -1 & -1 -1 +1 & +1 +1 -1 & -1 -1 +1
  \end{array}
  \]

- Coding by calculating the inner product $c_i s_i$ of the received signal and the chip $c_0 = - c_1$:

  \[
  \sum_{i=1}^{m} c_{0,i} s_i \quad \sum_{i=1}^{m} c_{1,i} s_i
  \]

- In the case of a superimposed signal, the original signal can be decoded by filter
- DSSS is used by GPS, WLAN, UMTS, ZigBee, Wireless USB based on the **Barker code**
  - Here for all $v<m$

  \[
  \left| \sum_{i=1}^{N-v} a_i a_{i+v} \right| \leq 1
  \]
  - Barker Code for 11Bit: +1 +1 +1 -1 -1 -1 +1 -1 -1 +1 -1
CDMA (Code Division Multiple Access)
- e.g. GSM (Global System for Mobile Communication)
- or UMTS (Universal Mobile Telecommunications System)

Uses chip-sequence with
- \( C_i \in \{-1,+1\}^m \)
- \(-C_i = (-C_{i,1}, -C_{i,2}, \ldots, -C_{i,m})\)

so that the normalized inner product for all \( i \neq j \) the result is 0.

\[
C_i \cdot C_j = \frac{1}{m} \sum_{k=1}^{m} C_{i,k} C_{j,k} = 0 .
\]

Synchronized recipients get a linear combination of A and B

Multiplying by the desired chip sequence yields the desired message.
CDMA: Example 1

- **Sender A:**
  - 0 = (-1,-1)
  - 1 = (+1,+1)

- **Sender B:**
  - 0 = (-1,+1)
  - 1 = (+1,-1)

- **A sends 0, B sends 0:**
  - Result: (-2,0)

- **C receives (-2,0):**
  - Decoding of A: (-2,0) • (-1,-1) = (-2)(-1) + 0(-1) = 2
  - A has therefore sent 0 because result is positive
CDMA: Example 2

- Sample-code:
  - Code $C_A = (+1,+1,+1,+1)$
  - Code $C_B = (+1,+1,-1,-1)$
  - Code $C_C = (+1,-1,+1,-1)$

- A sends Bit 0, B sends Bit 1, C sends nothing
  - $V = C_1 + (-C_2) = (0,0,2,2)$

- Decoding for A: $V \cdot C_1 = (0,0,2,2) \cdot (+1,+1,+1,+1) = 4/4 = 1$
  - results in Bit 0

- Decoding for B: $V \cdot C_2 = (0,0,2,2) \cdot (+1,+1,-1,-1) = -4/4 = -1$
  - results in Bit 1

- Decoding for C: $V \cdot C_3 = (0,0,2,2) \cdot (+1,-1,+1,-1) = 0$
  - results in: no Signal.
Repetition

- **Multiplexed**
  - Spatial Multiplexing
  - Frequency division multiplexing
  - Time division multiplexing
  - Code division multiplexing
  - Multiple-input multiple-output (next lecture)

- **Modulation**
  - Amplitude modulation
  - Phase modulation
  - Frequency modulation
Repetition:
Complex Numbers

- i: imaginary number with
  - $i^2 = -1$

- A complex number is a linear combination of a real part $a$ and imaginary $b$
  - $z = a + bi$

- Calculation rules:
  - $(a+bi)+(c+di) = (a+c) + (b+d)\ i$
  - $(a+bi) (c+di) = (ac - bd) + (ad + bc)\ i$
  - $1/ (a+b\ i) = (a-bi)/(a^2+b^2)$

- Complex conjugate
  - $(a+bi)^* = (a - bi)$
Exponentiation of Complex Numbers

- Important equation
  - $e^{i\pi} = -1$
  - $e^{i\varphi} = \cos \varphi + i \sin \varphi$

- Exponentiation of a complex number
  - $e^{a+bi} = e^a e^{bi} = e^a (\cos b + i \sin b)$

- Therefore
  - real part $e^{i\varphi}$: $\text{Re}(e^{i\varphi}) = \cos \varphi$
  - imaginary of $e^{i\varphi}$: $\text{Im}(e^{i\varphi}) = \sin \varphi$
Equivalent Representations of the FFT

- **Real number representation**
  - Sine and cosine functions of different frequencies

\[
g(x) = \sum_{k=0}^{N-1} a_k \cos \frac{2\pi kt}{T} + b_k \sin \frac{2\pi kt}{T}
\]

- **Complex representation**
  - Real part of the exponential function of different frequencies

\[
f(x) = \sum_{k=0}^{N-1} z_k e^{i2\pi kt/T}
\]

- **Computation of the inverse by cosine/sine integral product**

\[
a_k = \frac{2}{T} \int_{0}^{T} g(t) \cos(2\pi nt t) dt
\]

\[
b_k = \frac{2}{T} \int_{0}^{T} g(t) \sin(2\pi nt t) dt
\]

- **Computation of the inverse by the integral over the product with the complex conjugated carrier wave**

\[
z_k = \frac{1}{T} \int_{0}^{T} \left( e^{i2\pi kt/T} \right)^* f(x) dt
\]
\[ f(x) = \sum_{k=0}^{N-1} z_k e^{i2\pi kt/T} \]