Wireless Sensor Networks
2. Multiplexing

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Multiple Use of the Medium

- Spatial Multiplexing
  - Parallel and exclusive use of transmission channels
    - e.g. Extra lines / cells / directional antenna
- Frequency division multiplexing
  - Multiple signals to be transmitted in a frequency range of bundled;
  - In radio transmission, different frequencies are assigned to different stations.
- Time division multiplexing
  - Delayed transmission of multiple signals
- Code division multiplexing
  - Coding of the signal into orthogonal codes, which can now be broadcast simultaneously on one frequency
  - Decoding with overlay also possible
- Multiple-Input Multiple-Output
  - Sending and receiving antennas by several
  - Using the spatial and temporal information about location of several waves
    - e.g. 802.11n

Space

- Spatial distribution (space multiplexing)
  - Utilization of distance loss for the parallel operation of different radio cells → cellular networks
  - Using directional antennas for communications directed requested
  - GSM antennas with directional characteristics
  - Radio with a satellite dish
  - laser communications
  - infrared communication

Frequency Multiplexing

- Allocation of bandwidth in frequency sections
- Spread of the channels and hopping
  - Direct Sequence Spread Spectrum (DSSS)
  - Xor a signal with a pseudo-random number sequence at the transmitter and receiver (Relates to code-division multiplexing)
  - Other signals appear as background noise
- Frequency Hopping Spread Spectrum (FHSS)
  - Frequency change by pseudo-random numbers
  - two versions
    - Quick change (almost hopping): Multiple frequencies per user data bits
    - Slowly changing (slow hopping): Multiple user bits per frequency

Time Multiplexing

- Temporal distribution of sender-/receiver channel
- Participants receive exclusive periods (slots) on the media
- Accurate synchronization necessary
- Coordination necessary, or rigid division

Direct Sequence Spread Spectrum

- A chip is a bit sequence (given by {-1, +1}), which encode a smaller set of symbols
- E.g. Transmission signal: 0 = (+1,+1,-1), 1=(-1,-1,+1)

0	1	0	1
+1 +1 -1	-1 -1 +1	+1 +1 -1	-1 -1 +1

- Coding by calculating the inner product $c_i\, s_i$ of the received signal and the chip $c_0 = - c_1$:

$$\sum_{i=1}^{m} c_{0,i} s_i \qquad \sum_{i=1}^{m} c_{1,i} s_i$$

- In the case of a superimposed signal, the original signal can be decoded by filter
- DSSS is used by GPS, WLAN, UMTS, ZigBee, Wireless USB based on the **Barker code**
  - Here for all v<m

$$\left| \sum_{i=1}^{N-v} a_i a_{i+v} \right| \leq 1$$

- Barker Code for 11Bit: +1 +1 +1 –1 –1 –1 +1 –1 –1 +1 –1

Code Division Multiple Access (CDMA)

- CDMA (Code Division Multiple Access)
  - e.g. GSM (Global System for Mobile Communication)
  - or UMTS (Universal Mobile Telecommunications System)
- Uses chip-sequence with
  - $C_i \in \{-1,+1\}^m$
  - $-C_i = (-C_{i,1}, -C_{i,2}, \ldots, -C_{i,m})$
- so that the normalized inner product for all i ≠ j the result is 0.

$$C_i \bullet C_j = \frac{1}{m} C_i \cdot (C_j)^T = \frac{1}{m} \sum_{k=1}^{m} C_{i,k} C_{j,k} = 0 \; .$$

- Synchronized recipients get a linear combination of A and B
- Multiplying by the desired chip sequence yields the desired message.

CDMA: Example 1

- Sender A:
  - 0 = (-1,-1)
  - 1 = (+1,+1)
- Sender B:
  - 0 = (-1,+1)
  - 1 = (+1,-1)
- A sends 0, B sends 0:
  - Result: (-2,0)
- C receives (-2,0):
  - Decoding of A: (-2,0) • (-1,-1) = (-2)(-1) + 0(-1) = 2
  - A has therefor sent 0 because result is positive

CDMA: Example 2

- Sample-code:
  - Code C_A = (+1,+1,+1,+1)
  - Code C_B = (+1,+1,-1,-1)
  - Code C_C = (+1,-1,+1,-1)
- A sends Bit 0, B sends Bit 1, C sends nothing
  - $V = C_1 + (-C_2) = (0,0,2,2)$
- Decoding for A: $V \cdot C_1 = (0,0,2,2) \cdot (+1,+1,+1,+1) = 4/4 = 1$
  - results in Bit 0
- Decoding for B: $V \cdot C_2 = (0,0,2,2) \cdot (+1,+1,-1,-1) = -4/4 = -1$
  - results in Bit 1
- Decoding for C: $V \cdot C_3 = (0,0,2,2) \cdot (+1,-1,+1,-1) = 0$
  - results in: no Signal.

Repetition

- Multiplexed
  - Spatial Multiplexing
  - Frequency division multiplexing
  - Time division multiplexing
  - Code division multiplexing
  - Multiple-input multiple-output (next lecture)
- Modulation
  - Amplitude modulation
  - Phase modulation
  - Frequency modulation

Repetition: Complex Numbers

- i: imaginary number with
  - $i^2 = -1$
- A complex number is a linear combination of a real part a and imaginary b
  - $z = a + bi$
- Calculation rules:
  - $(a+bi)+(c+di) = (a+c) + (b+d)\, i$
  - $(a+bi)\,(c+di) = (ac - bd) + (ad + bc)\, i$
  - $1/\,(a+b\,i) = (a-bi)/(a^2+b^2)$
- Complex conjugate
  - $(a+bi)^* = (a - bi)$

Exponentiation of Complex Numbers

- Important equation
  - $e^{i\pi} = -1$
  - $e^{i\varphi} = \cos\varphi + i \sin\varphi$
- Exponentiation of a complex number
  - $e^{a+bi} = e^a e^{bi} = e^a (\cos b + i \sin b)$
- Therefore
  - real part $e^{i\varphi}$: $Re(e^{i\varphi}) = \cos\varphi$
  - imaginary of $e^{i\varphi}$: $Im(e^{i\varphi}) = \sin\varphi$

Equivalent Representations of the FFT

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- Real number representation
  - Sine and cosine functions of different frequencies

$$g(x) = \sum_{k=0}^{N-1} a_k cos \frac{2\pi kt}{T} + b_k sin \frac{2\pi kt}{T}$$

- Computation of the inverse by cosine/sine integral product

$$a_k = \frac{2}{T} \int_0^T g(t) \cos(2\pi n f t) dt$$

$$b_k = \frac{2}{T} \int_0^T g(t) \sin(2\pi n f t) dt$$

- Complex representation
  - real part of the exponential function of different frequencies

$$f(x) = \sum_{k=0}^{N-1} z_k \ e^{i2\pi kt/T}$$

- Computation of the inverse by the integral over the product with the complex conjugated carrier wave

$$z_k = \frac{1}{T} \int_0^T \left(e^{i2\pi kt/T}\right)^* f(x) dt$$

$$f(x) = \sum_{k=0}^{N-1} z_k \; e^{i2\pi kt/T}$$