ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Algorithms for Radio Networks

Localization

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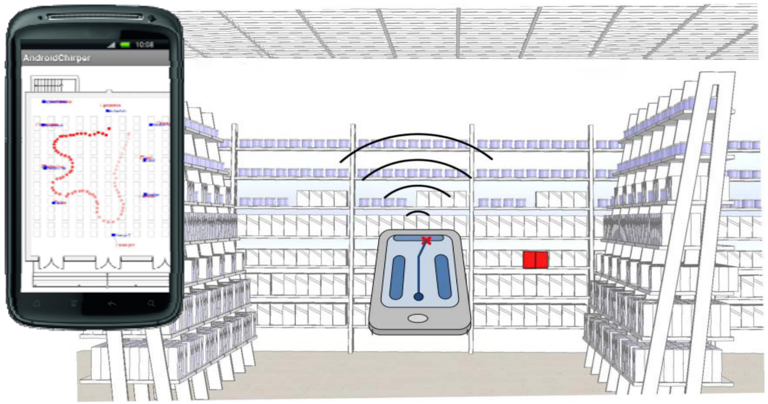
Localization

- Localization in an empty environment?
  - Requires some “stuff” around
  - Determine the physical position or logical location
- Reference points (“landmarks”)
  - Natural: Trees, mountains, river bend, earth’s surface, sun, stars, ...
  - Artificial: Road signs, Surveyor’s mark, Retro-reflector, buoys, lighthouse, radio beacon, ...
- Coordinate systems
  - Global coordinate frame, Earth coordinates
  - Local reference frame: Cartesian grid, floor tiles
  - Absolute or relative coordinates

Localization

- **Applications**
  - **Surveying, geodesy**
  - **Naval navigation, aviation, space flight**
  - **Navigation of people inside buildings in urban areas**
  - **Cars on roads, logistics**
  - **Navigation of robots: Autonomous mobile units**
  - **Industrial machines, tools: Drills, rivet hammers**
  - **Networks: Routing algorithms, sensor networks**
  - **...and many more!**

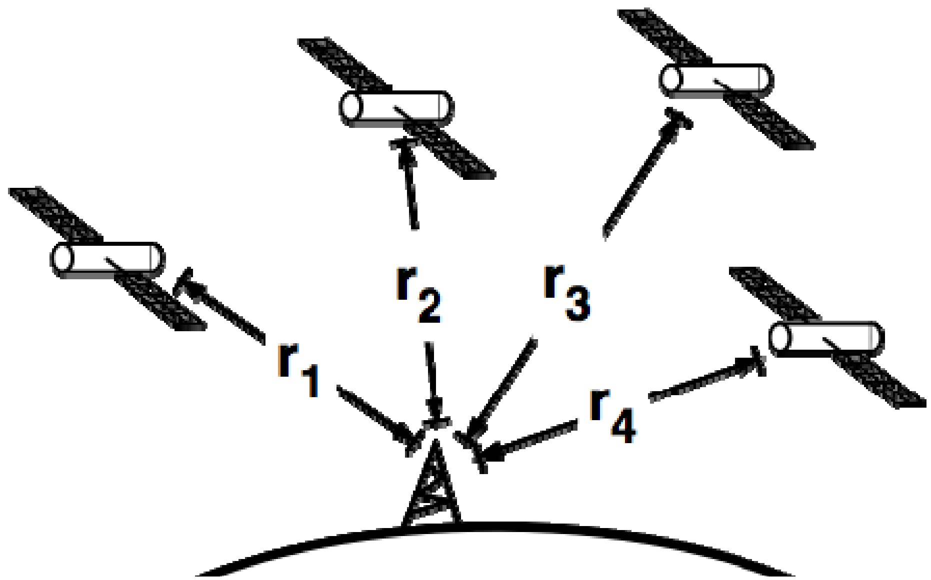


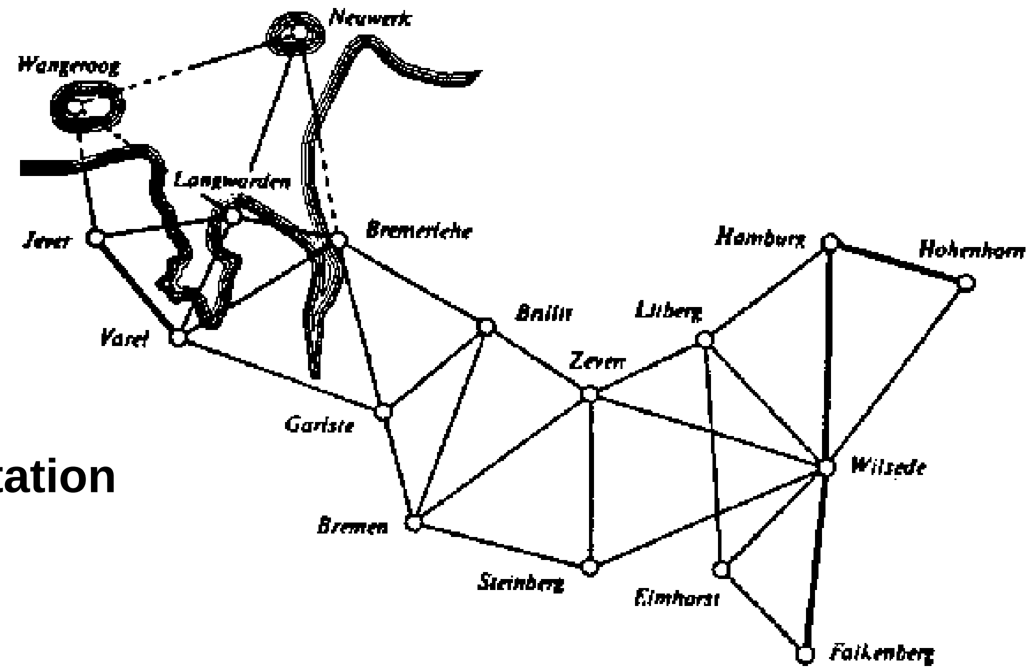
Localization

- **Parameter**
  - **Centralized or distributed computing**
  - **Availability of position information: Active vs. passive localization**
  - **Application**
    - **Indoors, outdoors, global**
  - **Sources of information: Sound, light, radio signal, magnetic field, ...**
- **Metrics**
  - **accuracy**
  - **precision**
  - **other costs**

Sources of Information

- Neighborhood information
  - Range provides coarse location information
    - e.g. GSM / UMTS cell, wireless IDs
- Triangulation and trilateration
  - Angle differences
  - distance measurement
- Analysis of the environment
  - Characteristic "signature" by radio conditions in the environment
- Inertial navigation systems
  - Measurement of acceleration and rotation



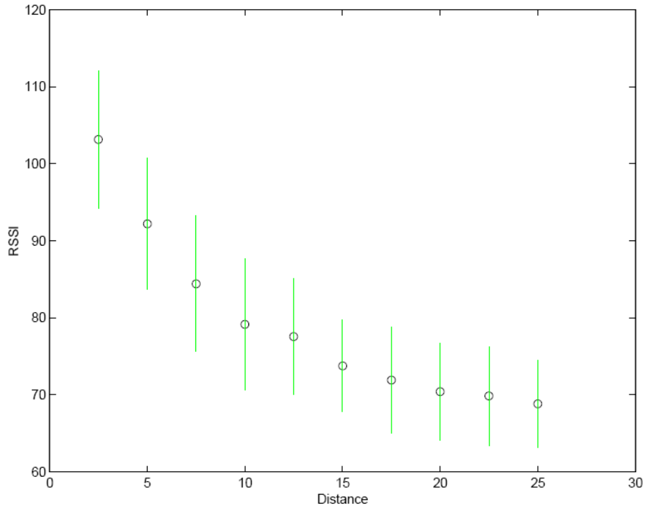


RSSI

- **Received Signal Strength Indicator**
  - **Using the path loss at a known transmission power**
  - **Measurement of the received signal**

$$P_{\text{recv}} = c\frac{P_{\text{tx}}}{d^\alpha} \Leftrightarrow d = \sqrt[\alpha]{\frac{cP_{\text{tx}}}{P_{\text{recv}}}}$$

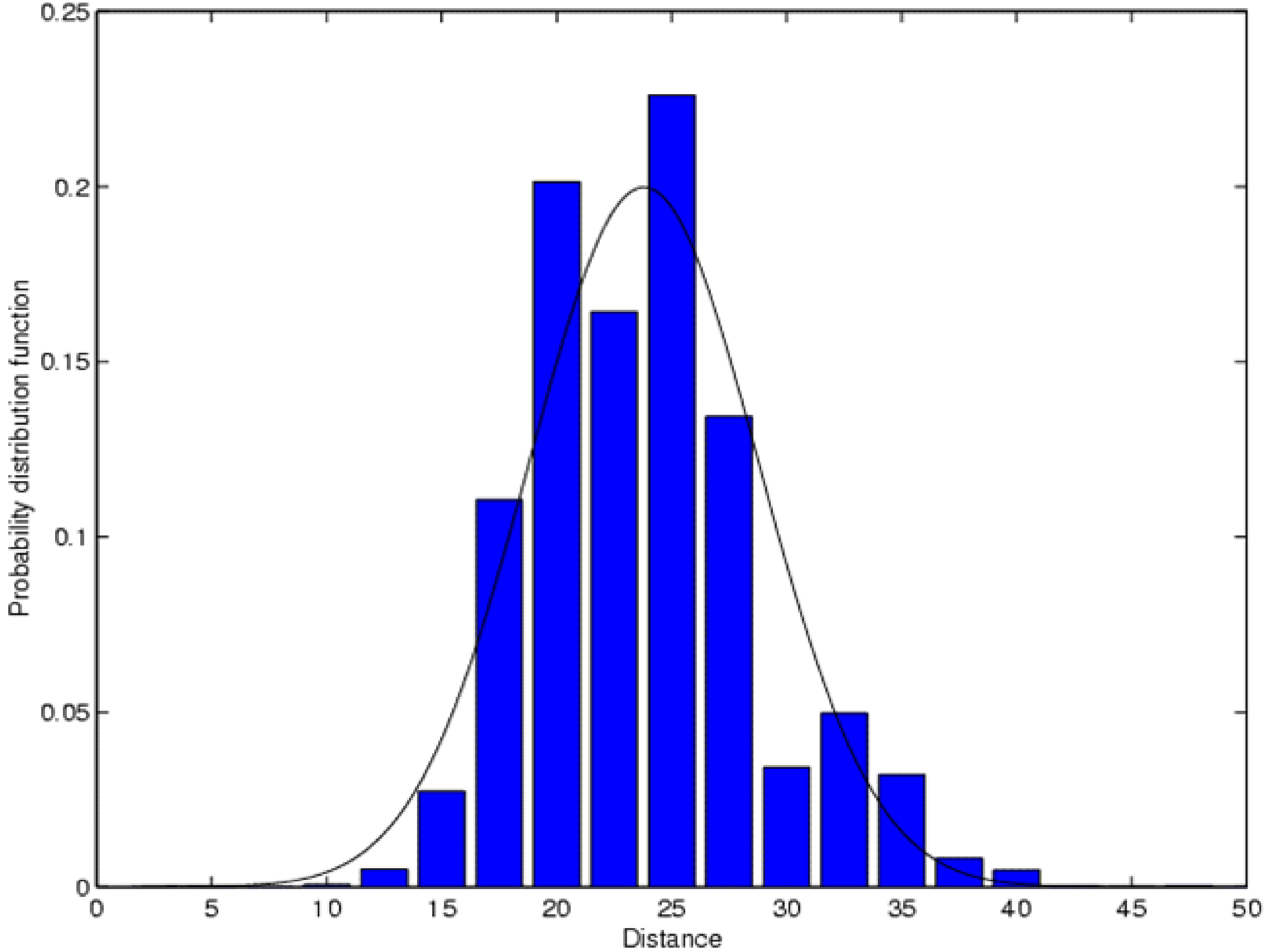
  - **Path loss exponent α, transmission power P_{tx}**
  - **Problem: High error rate**



[Sichitiu and Ramadurai, MASS 2004]

RSSI

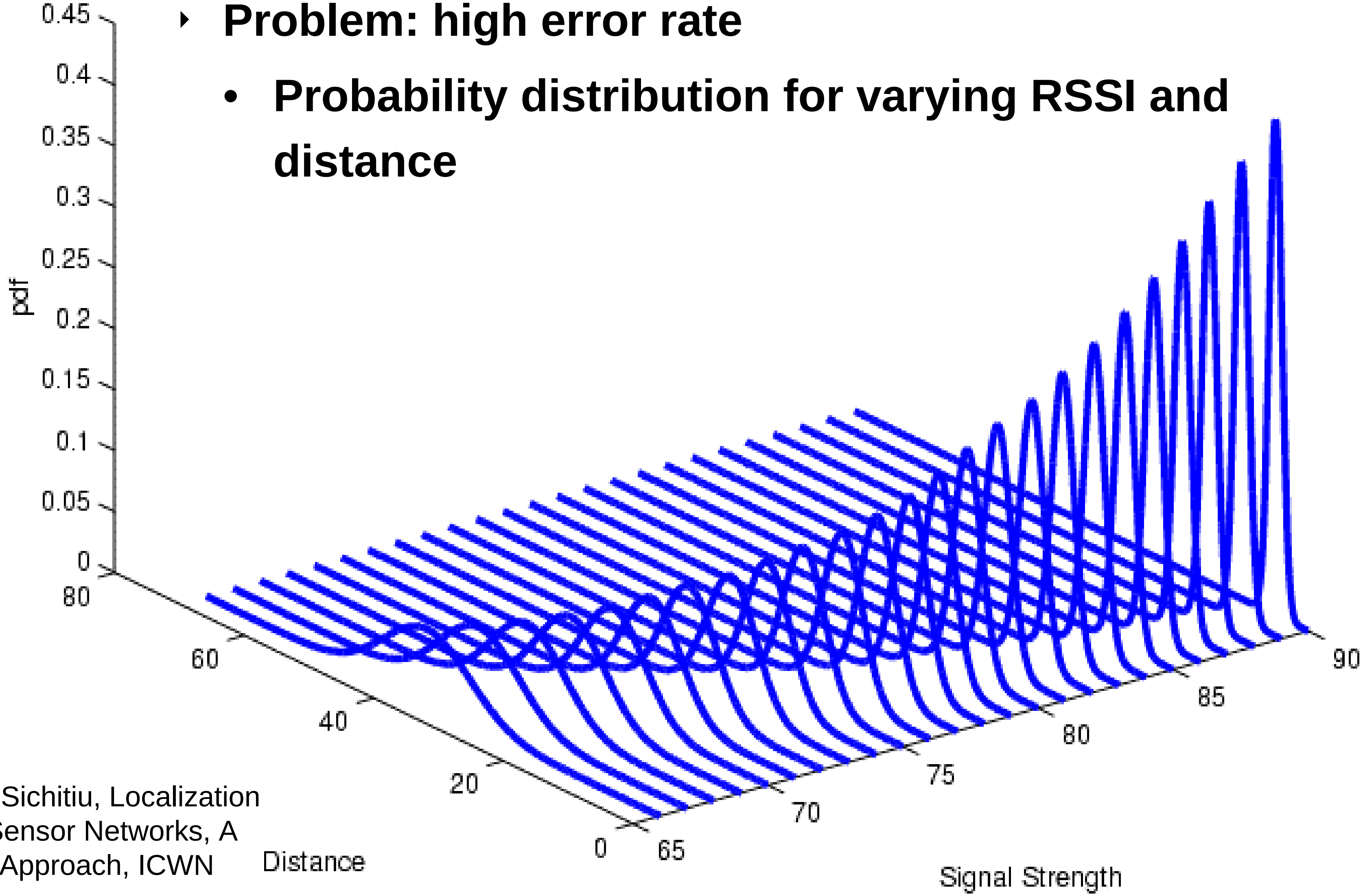
- **Problem: high error rate**
  - **Probability distribution for RSSI and given transmission power**



[Ramadurai, Sichitiu, Localization in Wireless Sensor Networks, A Probabilistic Approach, ICWN 2003]

RSSI

- Problem: high error rate
  - Probability distribution for varying RSSI and distance



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RSSI

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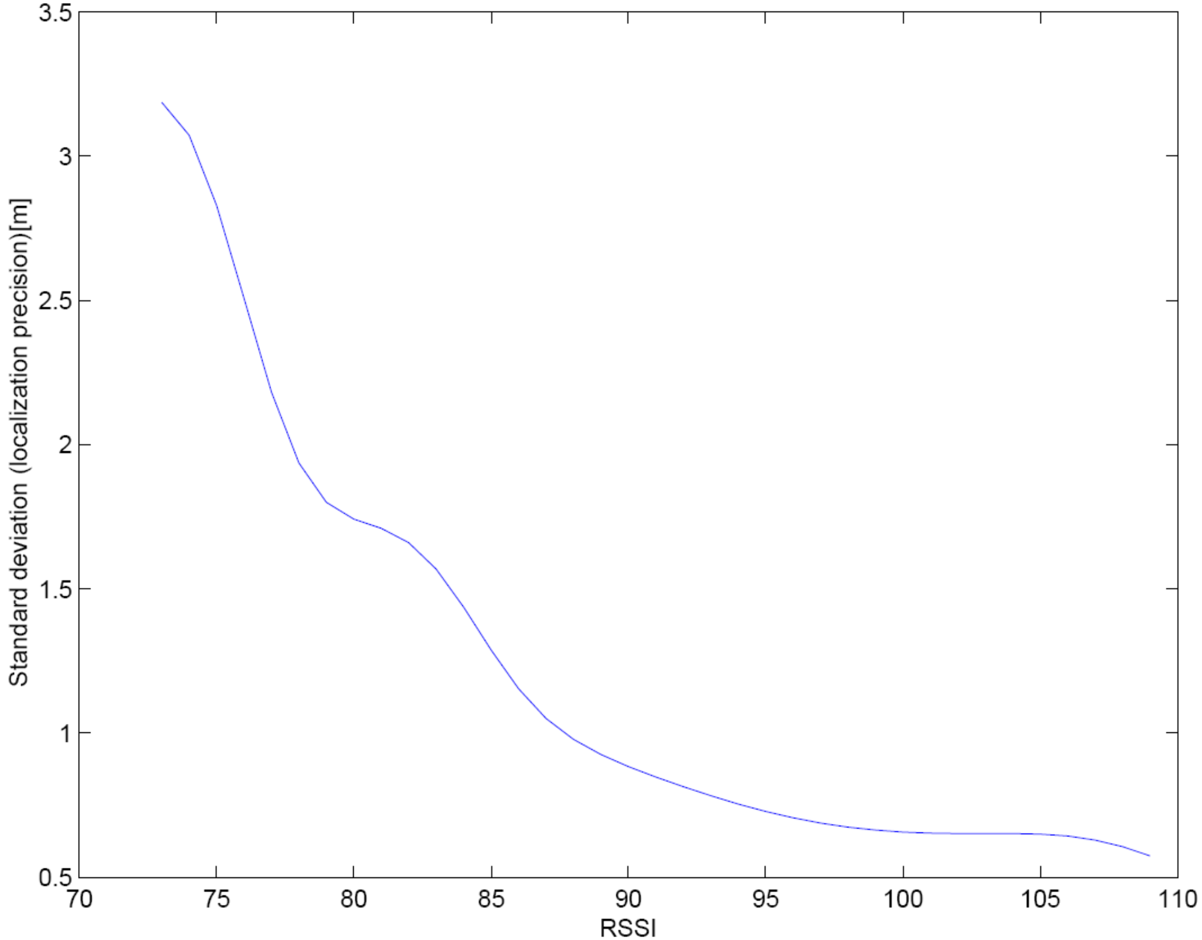


[Sichitiu and Ramadurai, MASS 2004]

Time of Arrival

- Time of arrival (TOA)
  - Transmission time ("Time of flight") is measured
  - Transmission time = Reception time – Send time
  - Results from the quotient:
    - Transmission time = distance / speed signal
- Problem
  - Positions of measurement points (anchors) must be known (usually...)
  - Accurate time measurement
  - Clock synchronization
  - Relative ranges require more anchors

Time *Difference* of Arrival (ToA)

- **Two different signals with different transmission speeds**
  - **E.g. ultrasound and radio signal, “thunderstorm”**
  - **Main component of the speed of sound**
  - **Calculate the different arrival times is distance**
  - **If one signal is very fast (e.g. “light”), eliminate it**
- **Problems:**
  - **calibration (hardware delay)**
  - **special hardware is required**

Round Trip time (ToA)

- Two way communication, send a signal back and forth between two transceivers
  - E.g. radio signal, sound signal
  - Distance = 1/2 * Round trip time / c
- Problems:
  - Again: calibration (hardware delay)
  - Requires two transmitters and two receivers
- Similar: Measure distance to an obstacle (reflection)
  - Distance measurement by Laser or ultrasound

Determination of Angles

- Optical angle measurement
  - done manually, sextant, theodolite
- laser beams
  - maximum accuracy
  - Controlled by rotating mirrors
- Directional antennas
  - free joint-directional or parabolic antennas
- Smart Antennae (antenna array)
  - (still) low precision (up to 1-2 degrees)
- Gyroscope

[Wikipedia]

Determination of Ranges

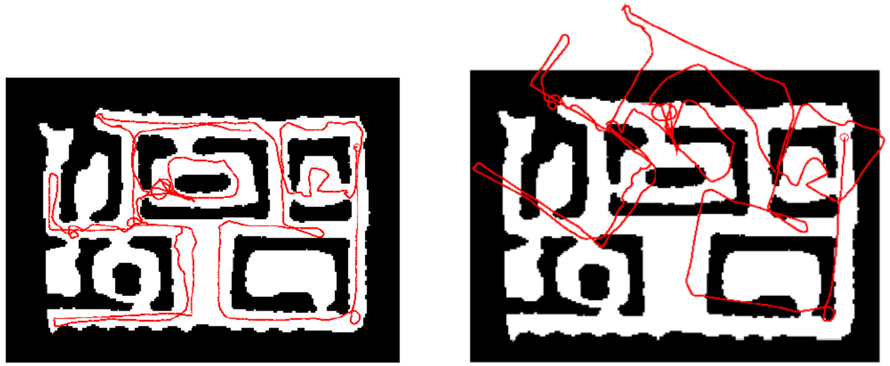
- Measuring tape
- Laser range finders: Measure phase shift
- Laser scanners: Depth imaging
- RF ranging: Radar
- Optical: ToF camera

[Würth, 2010]

[Sick, 2014]

Odometry

- **Measurement of travel distance**
  - **number of footsteps**
  - **odometer of a wheeled machine,**
  - **Mobile robot: Monitor individual wheels and steering angle**
  - **optical flow of vision / camera**
- **Integrate trajectory from a starting point ("dead reckoning")**
- **Problems:**
  - **Foot step size, wheel slip, different diameter of wheels**
  - **Error grows over time**



[AIS, University of Freiburg]

Coarse Localization Techniques

- **Hop-distance**
  - **in dense ad hoc networks or wireless sensor networks**
  - **approximate position by the number of hops to anchor points**
- **Overlapping connections**
  - **position at the intersection of the received transmission circuits**
- **Localization point in the triangle**
  - **determination of triangles of anchor points**
    - **in which the node lies**
  - **overlap provides approximate position**
- **“Fingerprinting” of signal strength measures**

Localization methods

- **Dead Reckoning: Relative localization depending on course and traveled distance**
- **Triangulation: Calculate the intersection of angular bearings**
- **Trilateration: Calculate the intersection of three range measurements (circles)**
- **Multilateration with *absolute* ranges: Calculate the intersection of *at least four* range measurements**
  - **In the plane: circles, in space: spheres**
  - **May be over-determined equation system**
- **Multilateration with *relative* ranges: Hyperbolic multilateration**
  - **Multilateration with unknown send time**
  - **Calculate intersection of hyperbolas / hyperboloids**

Dead Reckoning

- **Relative vector navigation, vectors of orientation** ϕ_i **and distance** d_i
- **Animals: “path integration” by special regions in hippocampus of desert ants (Wehner, 2003)**
- **Dead reckoning scheme:**

Dead Reckoning

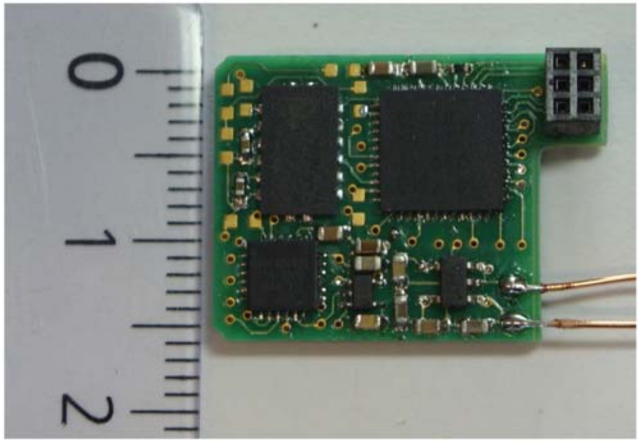
- **Example: Navigation of ships / airplanes**
  - **if course is known (compass)**
  - **if traveled distance is known (ship log, pitot tube)**
- **Prone to drift (water current, wind, wheel slip)**
- **Errors add up over time**

Inertial Navigation

- **Consider orientation and traveled distance as direction vector s_t at time t.**
- **What if only acceleration a_t is measured?**
  - ***Inertial navigation*, double integration**

$$\vec{s}(t) = \iint \vec{a}(t)dt^2 + \vec{s_0} + \vec{v_0} \cdot t$$

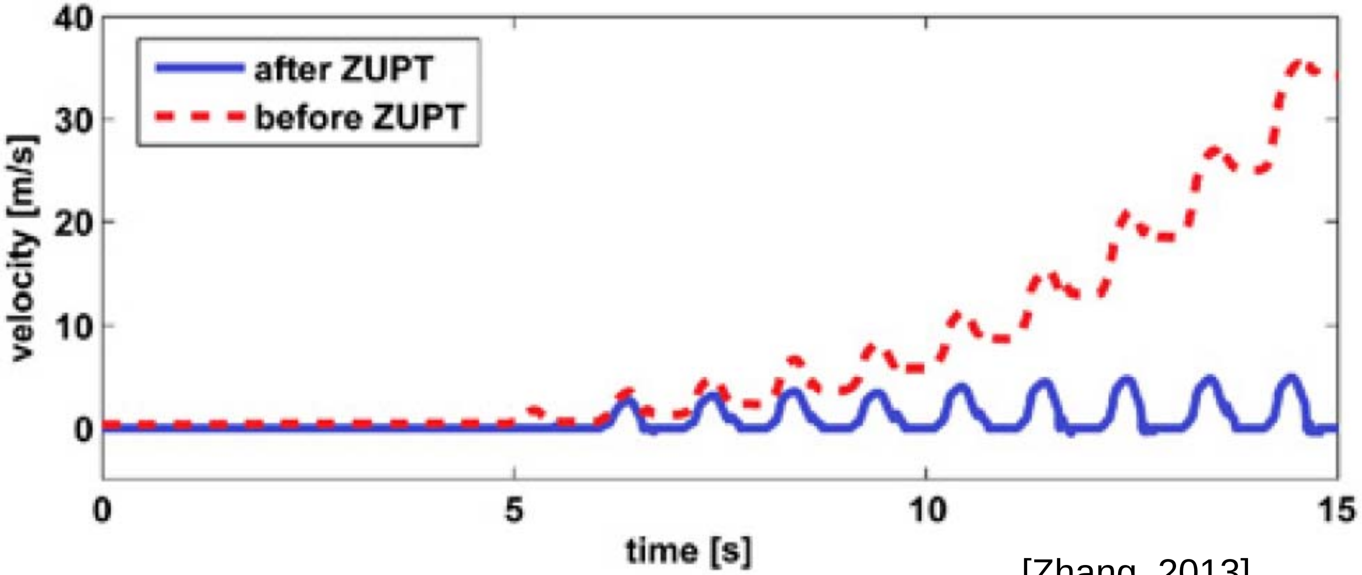
  - **Often also rotation is measured (angular velocity)**
- **Combine accelerometer, gyroscope, and compass:**
  - **Inertial Measurement Unit (IMU)**



[F. Höflinger, 2013]

Inertial Navigation

- **Foot-mounted MEMS-IMU**
  - **Errors add up over time**
- **Compensation: Zero velocity update**
  - **Detect footstep**
  - **Translation velocity is zero at this moment!**



[Zhang, 2013]

Triangulation

- Given a side of known length and two adjacent angles
- In the plane:
  - Calculate the intersection point of the other sides
  - Duality with trilateration: Triangle congruency (angle-side-angle) ←→ (side-side-side)
- On earth surface:
  - More complicated (spherical trigonometry)

Triangulation

- **Example: Navigation of ships / airplanes (cross bearing triangulation)**
  - **1) Bearings of two objects on a map**
    2) Time-shifted bearings of the same object

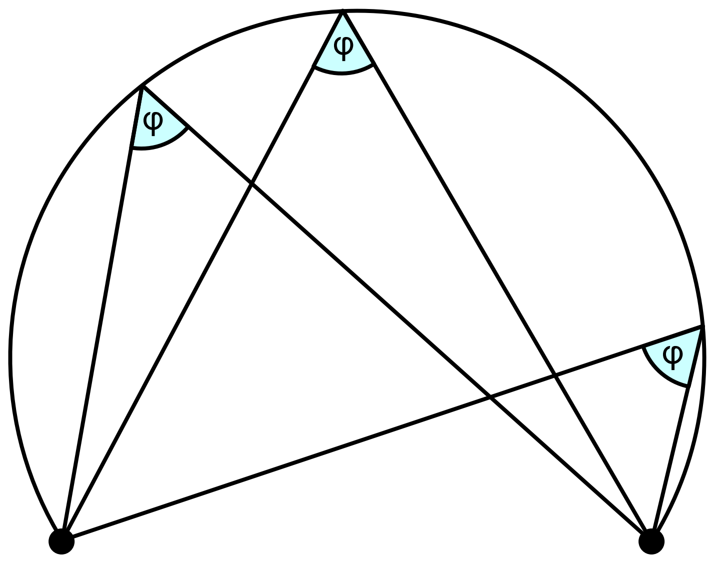
Triangulation

- Given a side of known length and the opposite angle
  - Triangle congruency: Does not define a triangle!
  - What else is possible?
- Given a lighthouse of known height h
  - Measurement of angle ϕ, use a sextant
  - Calculation of distance $d = h / \tan(\phi)$
  - Measurement of lighthouse bearing

    ➔ position in polar coordinates
- Height of lighthouse not known
  - Sail towards lighthouse

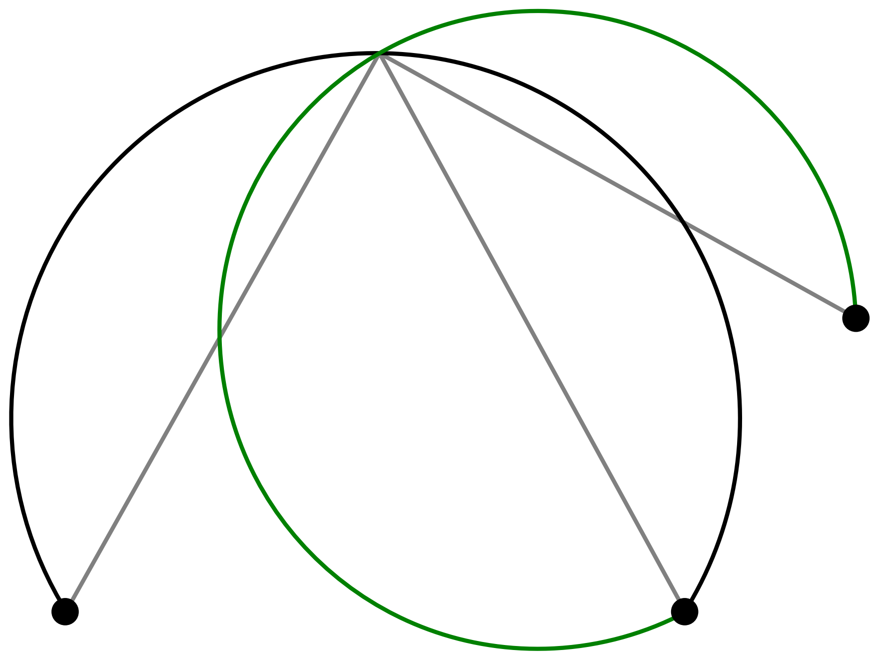
Triangulation

- **Given a side of known length and the opposite angle**
  - **Measure angle ϕ of two landmarks (by theodolite or by laser scanner)**
  - **If $\phi = 90^\circ$: Ship's position resides on Thales' circle**
  - **Other angles: generalization of Thales' circle**
  - **Circle of equal angles ("Fasskreisbogen")**



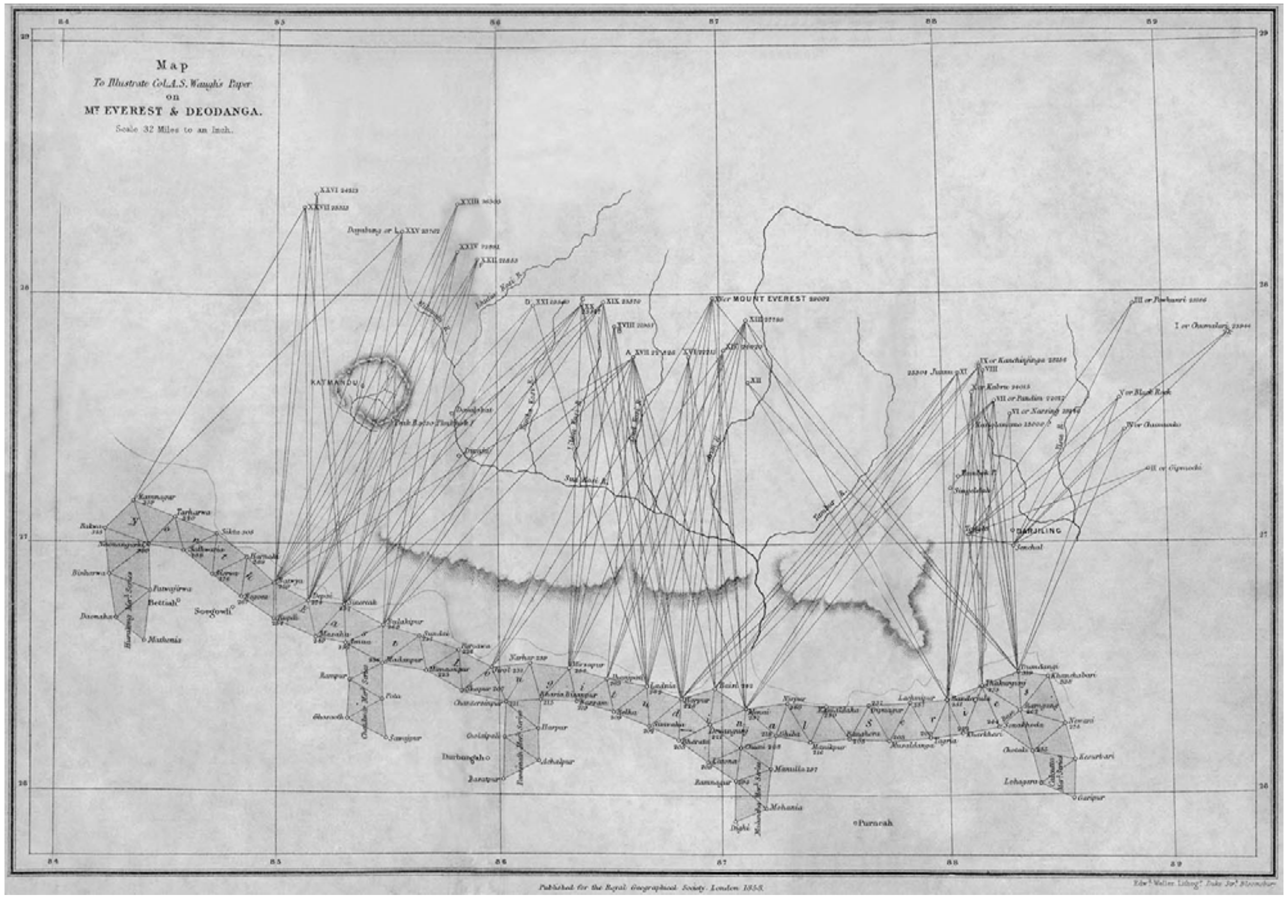
Triangulation

- **Given a side of known length and the opposite angle**
  - **Calculate position by a third landmark**



Triangulation

- **Height of Mt. Everest**
  - **8,840 m above NN (Sickdhar, 1856)**
  - **8,848 m (Survey of India, 1955)**
  - **8,850 m (GPS, 1999)**
  - **8,849 m (Radar reflectors, 2004)**
  - **...**



[A. Waugh, Mt. Everst & Deodanga, 1862.]

Trilateration

- **Assuming the distance to three points is given**
- **System of equations**
  - **(x_i, y_i): coordinates of an anchor point i,**
  - **r distance from the anchor point i**
  - **(x_u, y_u): unknown coordinates of a node**

$$(x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \text{ for } i = 1, \ldots, 3$$

- **Problem: Quadratic equations**
  - **Transformations lead to a linear system of equations**

Trilateration

- **System of equations**

$$(x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \text{ for } i = 1, \ldots, 3$$

- **Transformation**

$$(x_1 - x_u)^2 - (x_3 - x_u)^2 + (y_1 - y_u)^2 - (y_3 - y_u)^2 = r_1^2 - r_3^2$$
$$(x_2 - x_u)^2 - (x_2 - x_u)^2 + (y_2 - y_u)^2 - (y_2 - y_u)^2 = r_2^2 - r_3^2.$$

  - **results in:**

$$2(x_3 - x_1)x_u + 2(y_3 - y_1)y_u = (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2)$$
$$2(x_3 - x_2)x_u + 2(y_3 - y_2)y_u = (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2)$$

Trilateration as a Linear System of Equations

- **Forming a system of equations**

$$2\begin{bmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\ (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) \end{bmatrix}$$

- **Example:**
  - **$(x_1, y_1) = (2,1)$, $(x_2, y_2) = (5,4)$, $(x_3, y_3) = (8,2)$,**
  - **$r_1 = 10^{1/2}$, $r_2 = 2$, $r_3 = 3$**

$$2\begin{bmatrix} 6 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} 64 \\ 22 \end{bmatrix}$$

➔ $(x_u,y_u) = (5,2)$

Trilateration as a Linear System of Equations

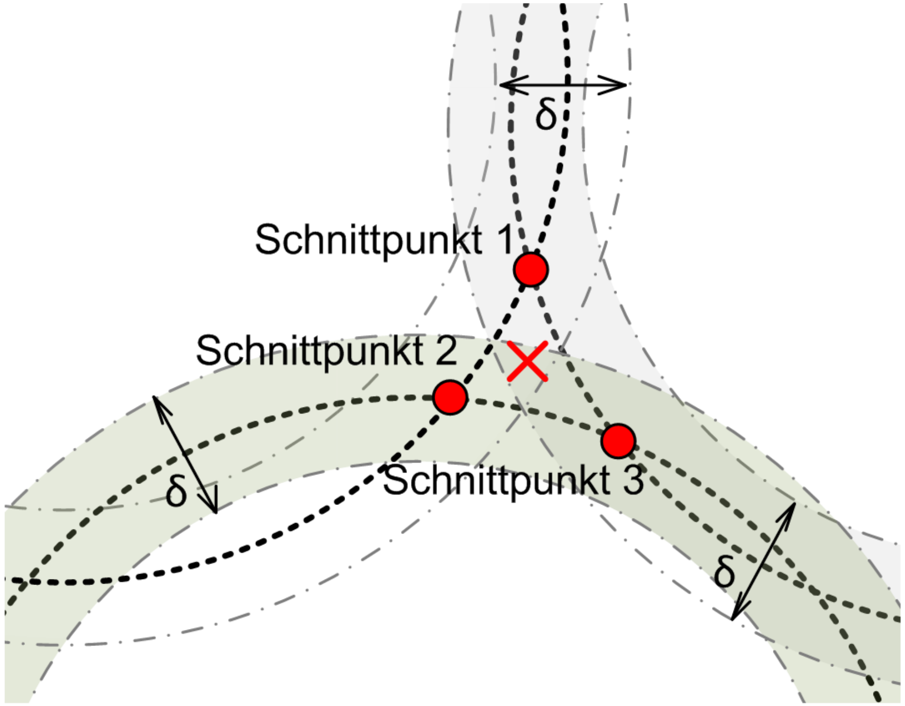
- **In three dimensions**
  - **Intersection of four spheres**

$$\underbrace{\begin{bmatrix}(d_1^2-d_4^2)-(x_1^2-x_4^2)-(y_1^2-y_4^2)-(z_1^2-z_4^2)\\(d_2^2-d_4^2)-(x_2^2-x_4^2)-(y_2^2-y_4^2)-(z_2^2-z_4^2)\\(d_3^2-d_4^2)-(x_3^2-x_4^2)-(y_3^2-y_4^2)-(z_3^2-z_4^2)\end{bmatrix}}_{\vec{b}} = 2\underbrace{\begin{bmatrix}(x_4-x_1)(y_4-y_1)(z_4-z_1)\\(x_4-x_2)(y_4-y_2)(z_4-z_2)\\(x_4-x_3)(y_4-y_3)(z_4-z_3)\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_{P1}\\y_{P1}\\z_{P1}\end{bmatrix}}_{\vec{x}}$$

- **Solve $\mathbf{Ax = b}$ ➔ $\mathbf{x = A^{-1}b}$**

Trilateration

- **In case of measurement errors**

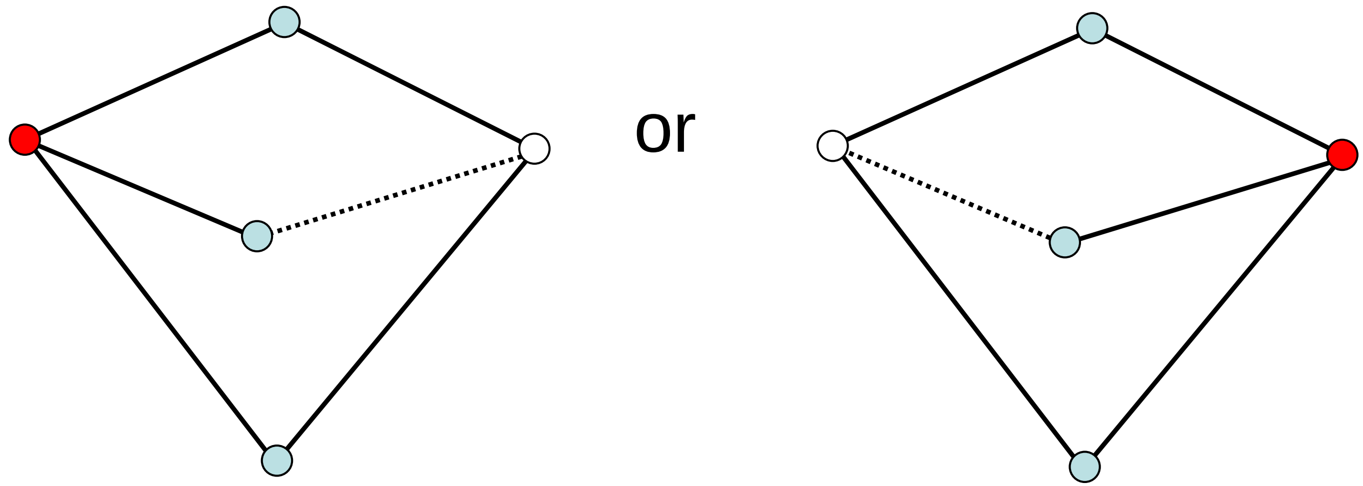


[F. Höflinger, 2013]

- **Averaging: e.g. centroid of triangle**

Trilateration

- **Measurement errors**
  - **Small distance errors can lead to large position errors**



- **flip ambiguity from noise**

Multilateration with *absolute* distances

- **Multilateration (absolute distances): Calculate the intersection of *at least four* distance measurements**
  - **May be over-determined equation system: More equations than variables**
  - **"No solution" in case of measurement errors**
- **Minimize sum of quadratic residuals: *Least squares***
- **Vector notation**
  - **Solve** $(\mathbf{A}^T\mathbf{A})\mathbf{x} = \mathbf{A}^T\mathbf{b} \Rightarrow \mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$
  - **Matrix inverse by Gauss-Jordan elimination**

Multilateration with *relative* distances

- **Multilateration (relative): Calculate the intersection of *relative* distance measurements**
  - **Emission time e unknown!**
  - **Measure only reception times T_i, $i = 1, ..., n$**
  - **System of equations $T_i = e + \| r_i - s \| / c$**
  - **...for a signal traveling from s to receivers r_i**
- **Subtract two absolute times T_i and T_j:**
  - $T_i - T_j = \| r_i - s \| / c - \| r_j - s \| / c =: \Delta t \quad (i, j = 1, ..., n)$
  - **System of hyperbolic equations**
  - **Time Difference of Arrival Δt, relative distance $\Delta d = c \Delta t$**

Multilateration with *relative* distances

- **Advantages**
  - **No cooperation of signal emitter**
  - **Hardware delays cancel out (both emitter and receiver)**
  - **Passive localization / natural signal sources**
- **Disadvantages**
  - **Larger number of unknown values: Position and time**
  - **Synchronization still (usually) required**

Anchor-free localization

- "Anchor-free localization":
  - Hyperbolic multilateration
  - Unknown emitters s_j, and unknown receivers r_i
- Advantages:
  - No need to measure receiver positions
  - Self-positioning by passive information from the surroundings
- Disadvantages:
  - Even larger number of unknown variables

Anchor-free localization

- **For absolute distances d_{ik}:**
  - **Solve** $\| \mathbf{r}_{\boldsymbol{i}} - \mathbf{s}_{\boldsymbol{k}} \| = d_{ik} \qquad (i, j = 1, ..., n\ ;\ \ k = 1, ..., m)$
  - **Problem of intersecting circles / spheres**
  - **Bipartite distance graph:** $G = \left(\{\mathbf{r}_{\boldsymbol{i}}\}, \{\mathbf{s}_{\boldsymbol{k}}\}, \{d(i, k)\}\right)$
  - **Minimum case closed-from solutions known** [Kuang, et al., ICASSP 2013]

Anchor-free localization

- **For *relative* distances $\Delta d_{ijk} = d_{ik} - d_{jk}$:**
  - **Solve $\| \mathbf{r}_i - \mathbf{s}_k \| - \| \mathbf{r}_j - \mathbf{s}_k \| = \Delta d_{ijk}$**
  - **Problem of intersecting hyperbolas / hyperboloids**
  - **Closed-form solutions only for larger problem sets**
    [Pollefeys and Nister, ICASSP 2008], [Kuang and Åström, EUSIPCO 2013]
  - **Minimum problem set: Iterative/recursive approximations**
    [Wendeberg and Schindelhauer, Algosensors 2012]

Anchor-free localization

- **Degrees of freedom**

$T_{ik} = e_{ik} + \| \mathbf{r}_i - \mathbf{s}_k \| / \mathbf{c}$

(e_{ik}, $\mathbf{r}_i$, $\mathbf{s}_k$ unknown)

signal sources	receivers 1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	3	3	3	3	3	3	3	3
3	5	4	3	2	1	0	-1	-2
4	7	5	3	1	-1	-3	-5	-7
5	9	6	3	0	-3	-6	-9	-12
6	11	7	3	-1	-5	-9	-13	-17
7	13	8	3	-2	-7	-12	-17	-22
8	15	9	3	-3	-9	-15	-21	-27
9	17	10	3	-4	-11	-18	-25	-32
10	19	11	3	-5	-13	-21	-29	-37
11	21	12	3	-6	-15	-24	-33	-42
12	23	13	3	-7	-17	-27	-37	-47

signal sources	receivers 1	2	3	4	5	6	7	8
1	0	2	4	6	8	10	12	14
2	3	4	5	6	7	8	9	10
3	6	6	6	6	6	6	6	6
4	9	8	7	6	5	4	3	2
5	12	10	8	6	4	2	0	-2
6	15	12	9	6	3	0	-3	-6
7	18	14	10	6	2	-2	-6	-10
8	21	16	11	6	1	-4	-9	-14
9	24	18	12	6	0	-6	-12	-18
10	27	20	13	6	-1	-8	-15	-22
11	30	22	14	6	-2	-10	-18	-26
12	33	24	15	6	-3	-12	-21	-30

$$G_{\mathbf{2D}} = 2n + 3m - nm - 3$$

$$G_{\mathbf{3D}} = 3n + 4m - nm - 6$$

Anchor-free localization

- **Minimum cases**

	2D	3D
general setting	**4 / 6**	**5 / 10** **6 / 7**
far-field setting	**3 / 3 (sync.)** **3 / 5 (unsync.)**	**4 / 6 (sync.)** **4 / 9 (unsync.)**

Minimum number of required receivers / emitters

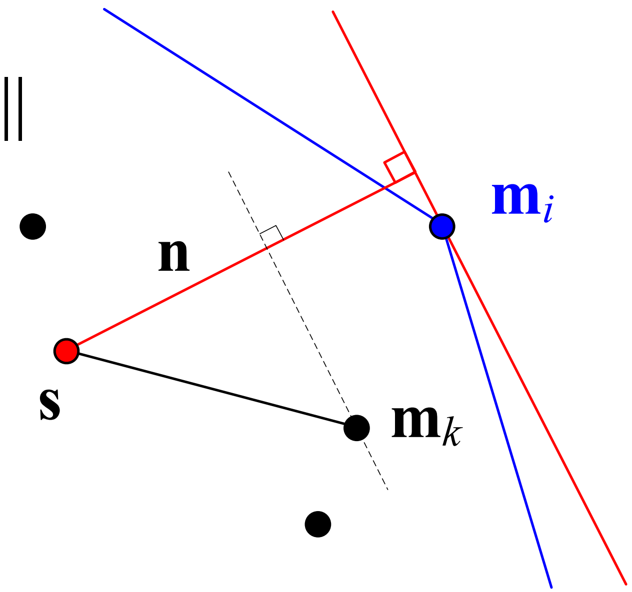
Anchor-free localization

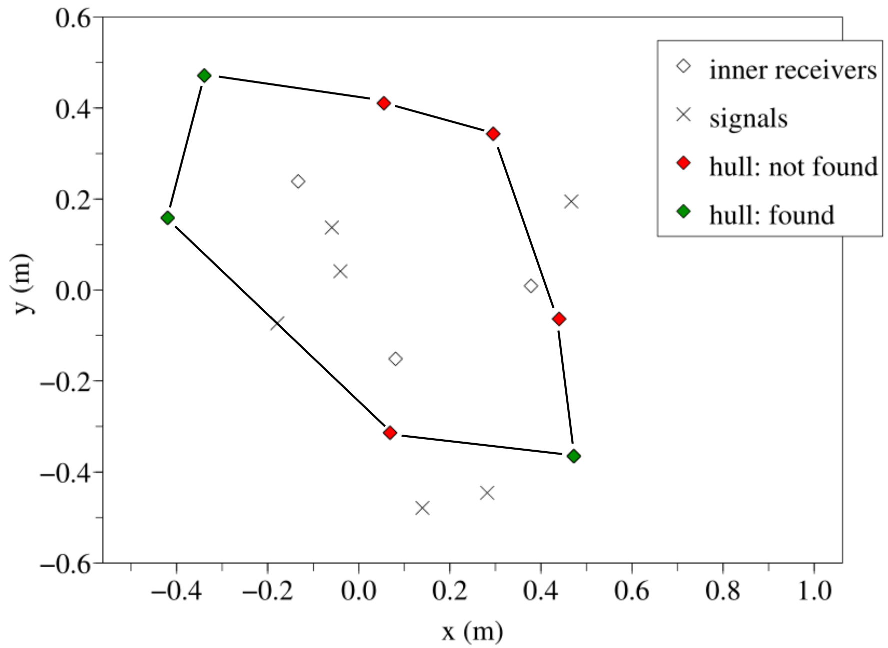
- **Strategies:**
  - **(1.) Estimate receiver topology from known information**
  - **(2.) Assume large number of emitters and receivers**
  - **(3.) Assume specific distribution of emitters and receivers**
  - **(4.) Heat the CPU: Optimization, branch-and-bound search, ...**

Anchor-free localization

- **(1.) Topology: Hull element**
  - **“The receiver which receives the last timestamp is an element of the convex hull”**

$$\mathbf{n}_0 = \mathbf{n} \,/\, \|\mathbf{n}\|$$



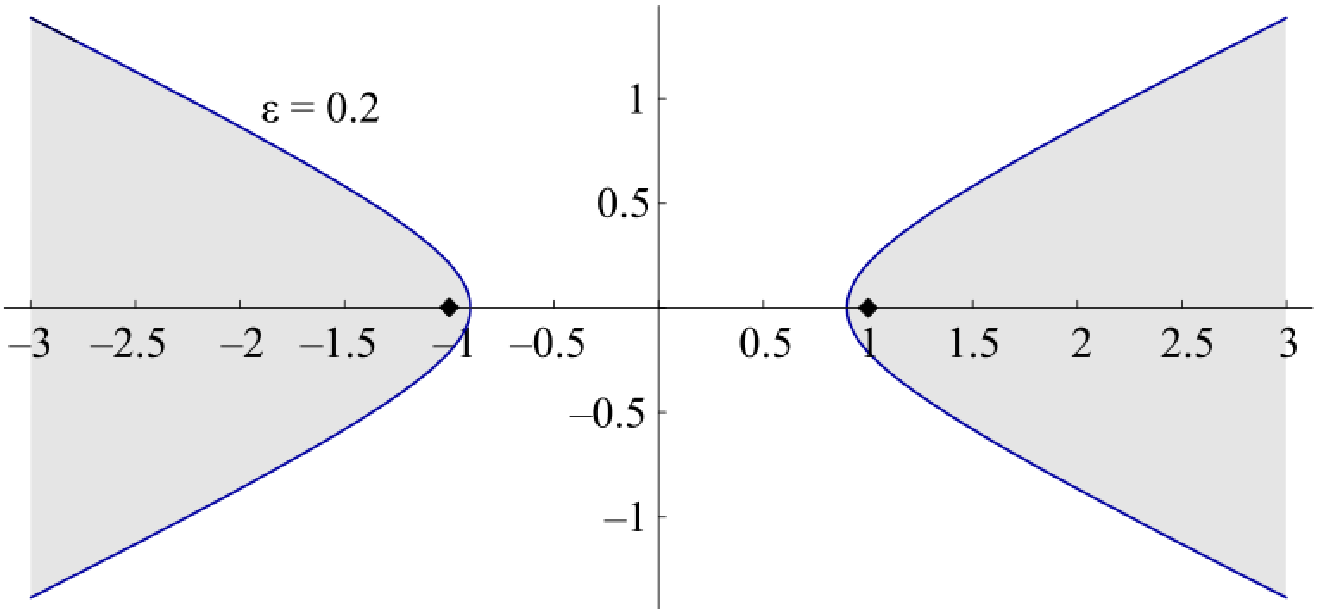


If exists i such that for all k: $T_i \geq T_k$, then holds:

$$(\mathbf{m}_i - \mathbf{s})^T \mathbf{n}_0 = \|\mathbf{m}_i - \mathbf{s}\| \geq \|\mathbf{m}_k - \mathbf{s}\| \geq (\mathbf{m}_i - \mathbf{s})^T \mathbf{n}_0$$

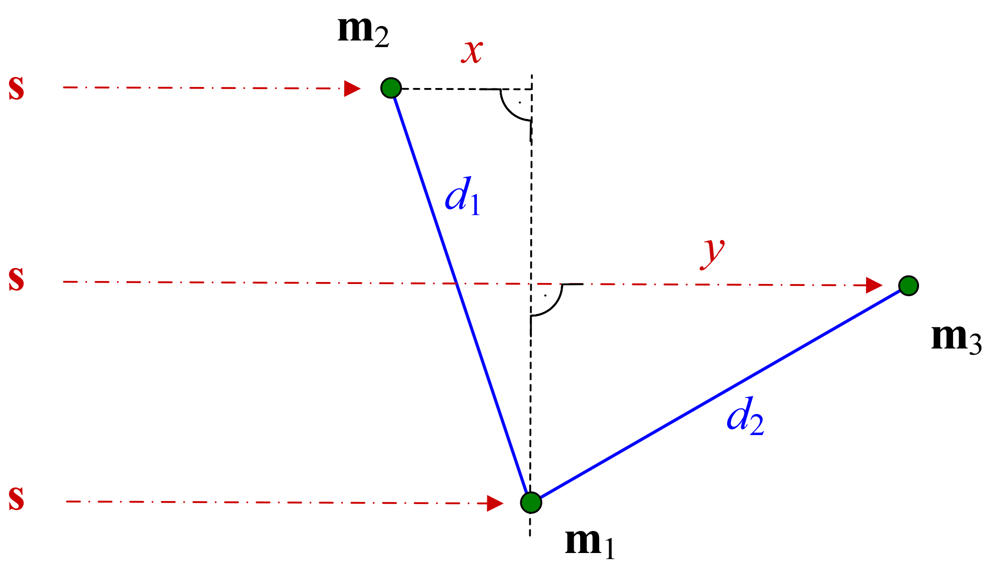
Anchor-free localization

- **(2.) Large number of signals: Statistical assumptions**
  [Schindelhauer, et al., SIROCCO 2011]
  - **Lemma: Many signals occur from the long side of any two receivers.**
  - **Estimate the distance:** $d \sim c/2\,(\Delta t_{max} - \Delta t_{min})$

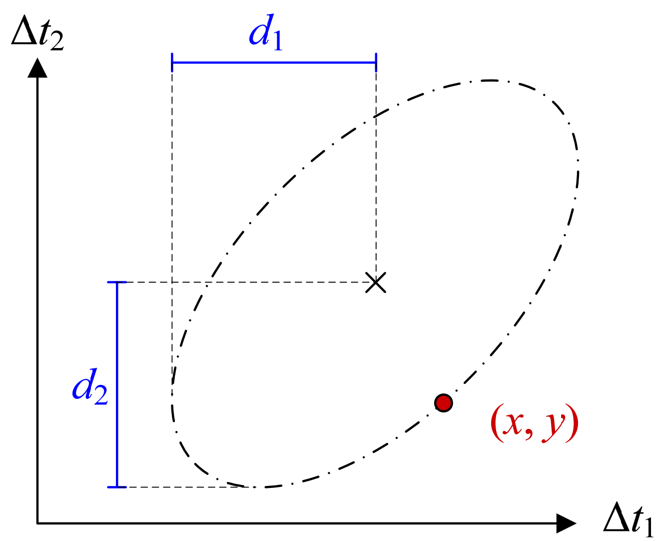


Anchor-free localization

- **(3.) Assume that signals occur from far away:**
  - **"far-field assumption", linear frontier of signal propagation**
- **The Ellipsoid TDoA Method** [Wendeberg, et al., TCS, 2012]
  - **Time differences of *three* receivers form an ellipse**



top-down view

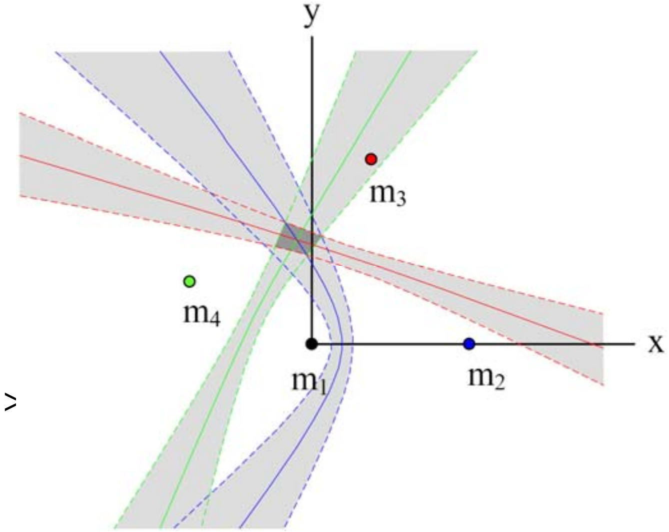


time differences

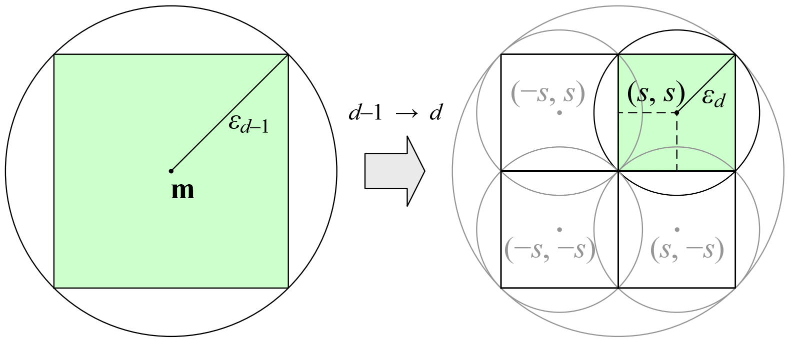
Anchor-free localization

- **(4.) Two-phased *branch-and-bound* algorithm in 2D**
  [Wendeberg and Schindelhauer, ALGOSENSORS 2012]

1. **"Bound": Test sub-problems if feasible up to error $\varepsilon \sim s$ with regard to measure-ments Δt_{ij}. Satisfy**
   $| \, \| m_i - s_j \| - \| m_1 - s_j \| - \Delta t_{ij} \, | \leq \varepsilon$ $(i >$
   or discard sub-problem

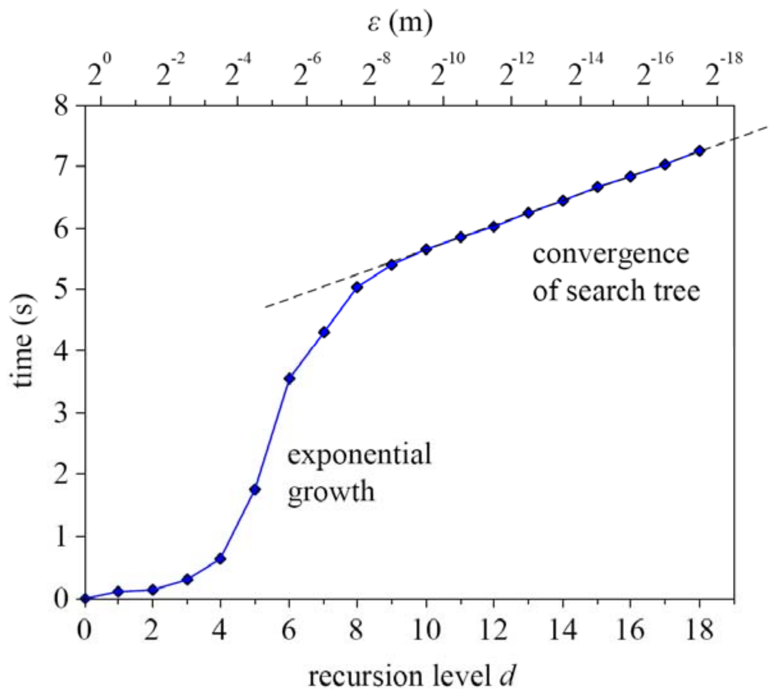


2. **"Branch": Divide feasible problems of size s^n into sub-problems of size $(s/2)^n$**



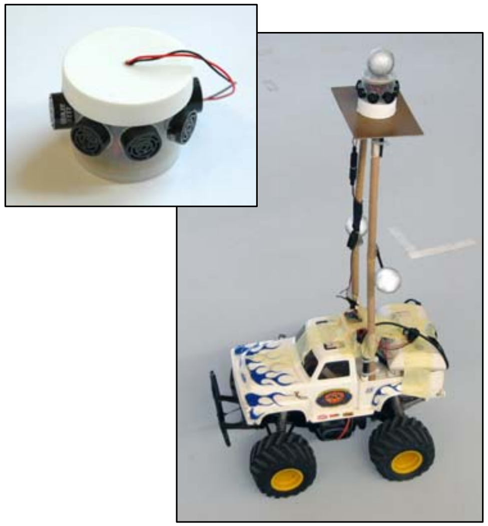
Anchor-free localization

- **Numeric simulation**
  - **Solution always found up to bound ε**
  - **In case of measurement errors: Solution up to $\varepsilon_{\text{tdoa}}$**
- **Behavior of search tree**
  - **Breadth-first search**
  - **Exponential growth / convergence of search tree**
  - **Runtime:** $\mathcal{O}((\sqrt{2}/\epsilon)^{2n-3}mn^2)$
- **➔ Minimum case FPTAS to Calibration-free TDoA**



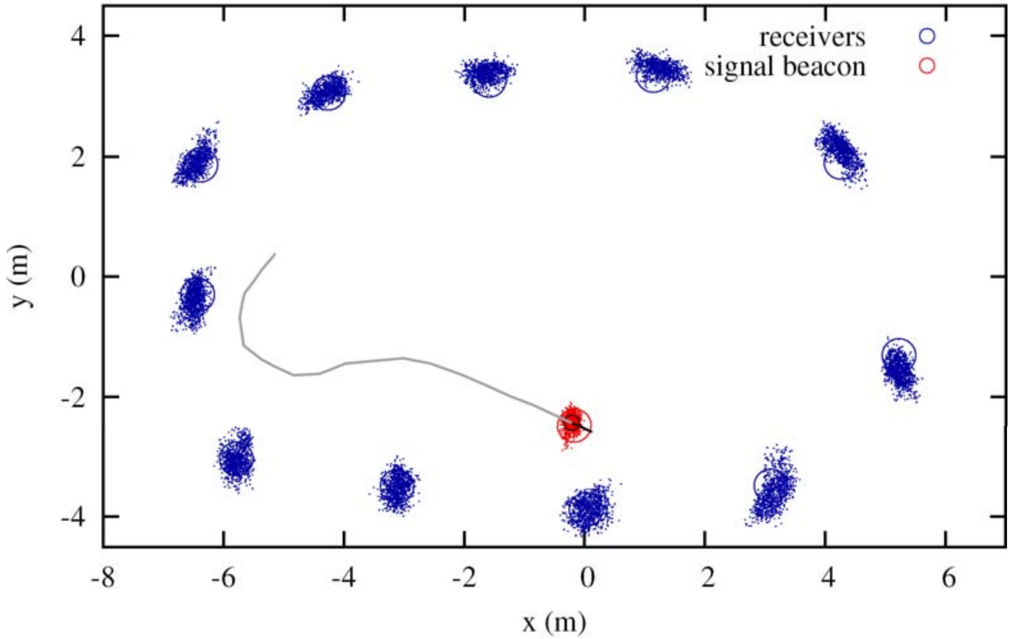
“Calibration-Free Tracking System”

- **Anchor-free TDoA Ultrasound Tracking System**
  [Wendeberg, Höflinger, Schindelhauer, and Reindl, LBS, 2013]
  - **In collaboration with IMTEK / Lab. for Electrical Instrumentation (EMP)**
  - **40 kHz ultrasound moving transmitter and fixed receivers**
  - **Receivers synchronized in a Wi-Fi network**

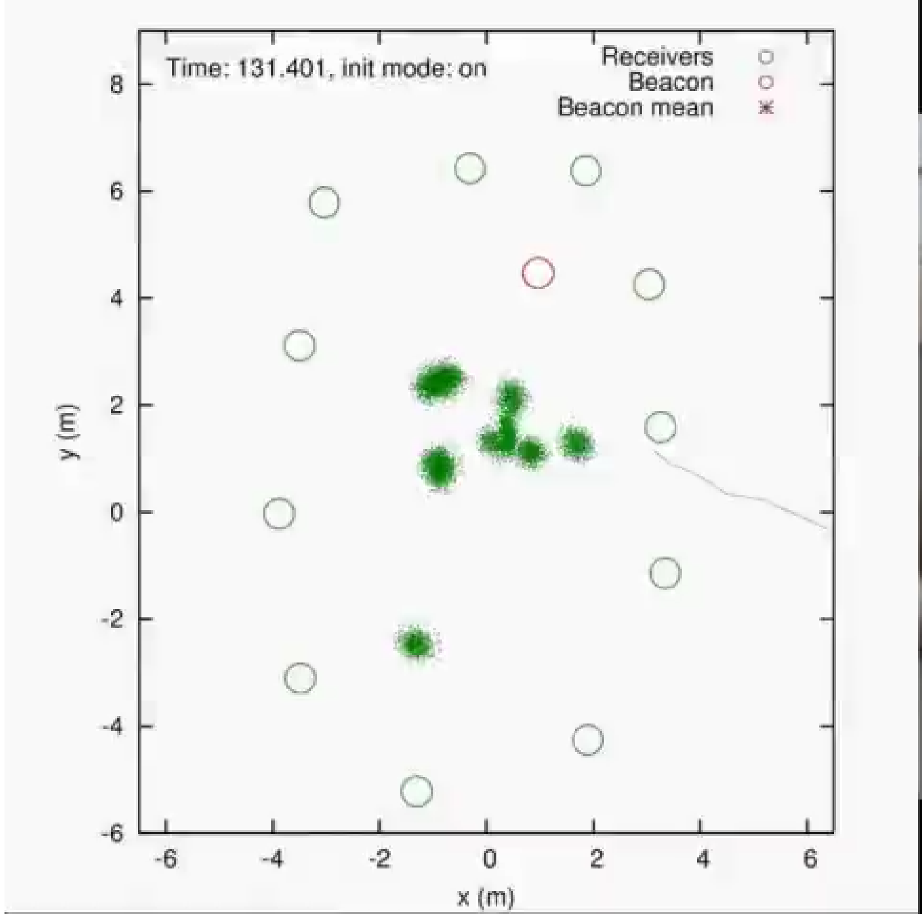


“Calibration-Free Tracking System”

- Tracking system is “calibration-free”
  - Arbitrary placement of ultrasound receivers
  - Compute positions of receivers by TDoA measures
  - Precision of ~ 5 cm



"Calibration-Free Tracking System"



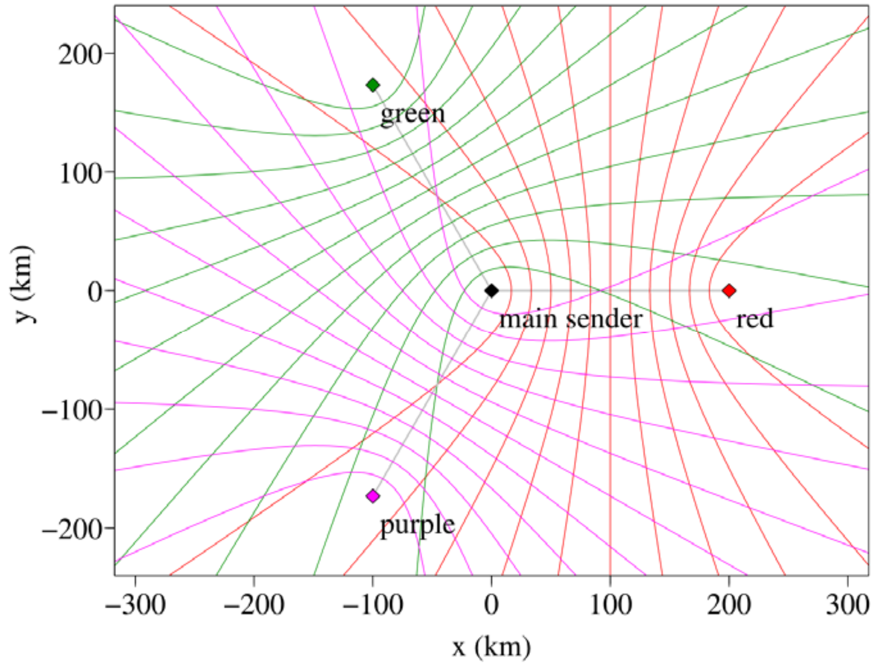
You Tube http://www.youtube.com/watch?v=V85wejcYyXs

Some More Available Localization Systems

- **Land stations**
  - **Decca**
  - **LORAN-C**
  - **Mobile cells**
  - **WLAN identification**
- **Satellite-based**
  - **NAVSTAR-GPS**
  - **GLONASS**
  - **Galileo**

Decca

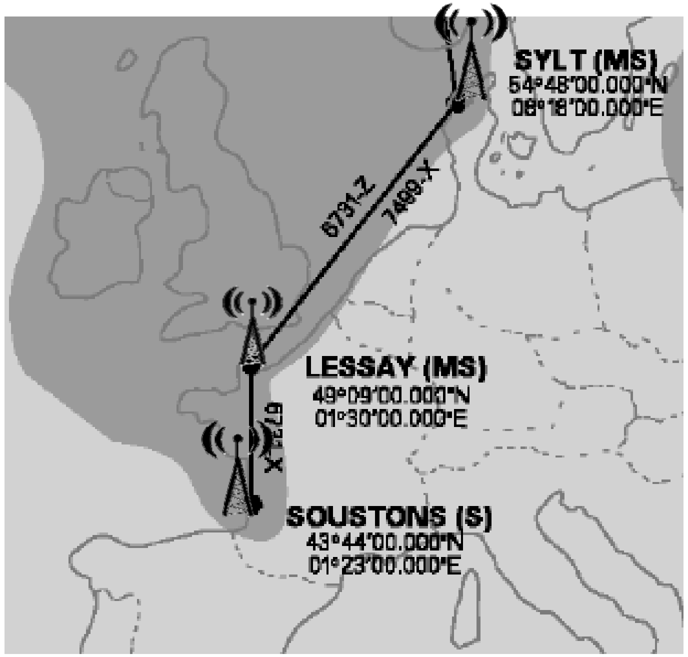
- W. O'Brien, Decca navigation system, ca. 1942 – 2000
- Hyperbolic multilateration
  - One main sender
  - Three slave senders (distance 100 – 200 km)
  - Senders synchronized
- TDoA by phase difference of continuous harmonics, e.g. {6*f*, 5*f*, 8*f*, 9*f* }, *f* = 14.167 kHz



- Point of departure must be known! (periodic phases)
- Range ca. 400 – 700 km, precision ca. 0.05 – 1 km

LORAN-C

- LOng RANge navigation system, 1957 – now
- Hyperbolic multilateration
  - Chains of senders (distance 100+ km)
- TDoA of discrete pulses of 100 kHz, identification of senders by CDMA (no overlap)
- Range up to 1,000 km, precision 0.01 – 0.1 km



[Wikipedia]

GNSS: GPS (I)

- **Global Positioning System (GPS), US Dpt. of Defense, since 1985, no “selective availability” since 2000**
- **24+ GPS satellites**
  - **earth orbit 20,000 km**
  - **send ephemerides (trajectory data) and atomic clock time**
  - **Frequency: 1.228 / 1.575 GHz**
- **GPS receiver**
  - **measures TDoA of satellite messages (by correlation)**
  - **has no precise clock!**
  - **calculates “pseudoranges”, 3D coordinates and time**
  - **requires at least 4 satellites (more is better)**

GNSS: GPS (II)

- GPS requires line-of-sight: No signal in forest, dense urban areas, indoors
- Precision: 5 – 15 m (good signal)
- Differential GPS
  - Reference receiver, compensating for atmospheric disturbances, precision up to 0.1 m
  - Modern geodetic systems: Even millimeters!

GNSS: GLONASS

- **GLONASS, russian GNSS, since 1993 (25 satellites)**
- **Technology similar to NAVSTAR-GPS**
- **Limited operation: in 2001 only 7 satellites alive, in 2011 available again (ca. 24 satellites)**
- **Loss of 3 satellites each in Dec. 2010 and in July 2013**
- **Supported by modern smart phones (Nokia Lumia series, Samsung Galaxy series, Apple iPhone 4S and later, and others)**

GNSS: Galileo

- **Galileo, european GNSS, adopted in 2008**
- **Technology similar to NAVSTAR-GPS**
- **Up to 30 satellites planned**
- **Availability expected for 2014 with 18 satellites**

Possible Improvements

- **Combination of different methods**
  - **magnetic field**
  - **air pressure**
  - **sonar**
- **Kalman filter**
  - **Extension of Markov filters**
- **Motion sensors**
  - **gyroscopes**
  - **acceleration sensors**

ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Algorithms for Radio Networks

Localization

University of FreiburgTechnical Faculty
Computer Networks and Telematics
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