Robust Tracking of a Mobile Receiver using Unsynchronized Time Differences of Arrival

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***Abstract*—Localization based on the time difference of arrival (TDoA) has turned out to be a promising approach for indoor environments, especially in combination with innovative self-calibrating TDoA algorithms that eliminate the need to measure the positions of reference receivers. We consider the previously unsolved problem of locating a moving target receiver by discrete signals from stationary beacons at unknown locations. We assume that the beacons are small and inexpensive and they require no further communication, i.e. they are unsynchronized. They only emit short discrete signals at regular intervals, of which we assume that they can be distinguished. The moving target travels on an unknown trajectory, receiving signals from the beacons and calculating the TDoA of the signals. First, we discuss adaptions of two TDoA algorithms by which the senders can be located from unknown signals. Second, we propose two novel approaches based on probabilistic state estimation to enable robust localization of the mobile receiver using the discrete arrival times, once the senders have been located. The probabilistic algorithms use the particle filter and the unscented Kalman filter to estimate the position and velocity of the target, as well as the unknown synchronization offsets of the senders. We provide a motion model and a sensor model for which we take into account that the signals of the beacons are received as singles, each at a different time. We verify the feasibility and robustness of our approach in extensive simulations, where we analyze the reliability of localization and compare both algorithms.**

I. Introduction

The rise of mobile technology in every-day life and the employment of mobile autonomous agents in industrial logistics has led to an increasing demand for location based services in indoor scenarios. Navigation satellite based solutions lack connectivity in these environments, so alternatives must be considered. One approach that prevailed in such environments is the time difference of arrival (TDoA) method using ultrasound. In contrast to many TDoA localization systems that track the position of a moving *signal emitter* [1], [2], we consider the problem of calculating and tracking the position of a continuously moving *receiver* by the signals of an arbitrary number of stationary sender beacons.

In this context, one may think of an application for industrial logistics where a large number of autonomous agents transport goods based on dynamic assignments. Forwarding goods from a material source to a drain requires absolute localization in the work environment. We suggest an application where inexpensive ultrasound beacons are randomly placed at the ceiling of a hall, emitting energy-efficient short signal bursts, and mobile autonomous units equipped with a cost-efficient ultrasound receiver unit, navigating by the locality information of the ultrasound system. By the agents passively receiving ultrasound signals from several beacons, the number of agents is not limited by the channel, in contrast to a scenario where agents actively emit an ultrasound signal.

A fundamental aspect of this setting is the assumption that signals can be clearly identified and distinguished by the receiver. This is achieved by modulating individual digital identification codes on the signal by the senders, which are detected by the receivers. A promising approach for this was demonstrated in [3]. The risk of collision of signals is high in such as setting, however loss of a few signals is acceptable. We assume that collisions can be detected and the signals are discarded.

In our setting, we assume that the beacon positions and the receiver to track are unknown in advance. Neither the sending times nor the intervals, when the senders emit new signals, are known to the receiver. As the receiver moves continuously, the mathematical problem results in an increasing number of unknown variables in each measurement, growing faster than constraints in the equation system are generated by the measurements. This equation system does not yield a unique solution in non-linear optimization approaches.

Furthermore, we assume that the sender clocks are unsynchronized, i.e. the time differences of the first signal of the beacons are not determined. Therefore, once the sender positions and the intervals are known, there still remains an unknown offset. Besides, in practical application appears the problem of oscillator drift of the sender clocks, which requires continuous observation and correction of the sender offsets, i.e. the time differences of the sender clocks.

In this paper we propose an approach based on probabilistic state estimation which recursively estimates the receiver position and the sender offsets only by the reception times of the signals from the sender beacons. We have developed a motion

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model for the moving receiver and a sensor model for the observed reception times which considers the unknown time offsets of the senders. We implemented the approach using a particle filter, yielding a robust solution, yet at high computational cost, and the unscented Kalman filter (UKF), creating an algorithm which is still reliable, and also computationally efficient. We evaluate and compare these two algorithms for which we have created and run numeric simulations.

This paper is composed of six sections. After discussing some related approaches in Section II we introduce the problem in Section III. In Section IV two methods to localize the senders are discussed, as the senders are unknown in the beginning. Section V describes the proposed approaches with the particle filter and the unscented Kalman filter and the motion and sensor models. The algorithms are analyzed and compared in Section VI where we present simulation results in a typical application scenario.

II. Related work

In mobile robotics the localization based on landmarks is a well-known approach. Based on these measurements they estimate their position and velocity. For robust state estimation probabilistic estimators are applied, such as extended Kalman filters [4] and combinations of the unscented Kalman filter and the unscented particle filter [5].

Saloranta and Abreu present a solution [6] in which a moving vehicle is able to localize the surrounding senders and itself using a weighted least squares procedure with the knowledge of the trajectory and odometry measurements. This procedure is limited by its assumption of a simple trajectory. The method used by Chan and Wen [7] is based on the Angle of Arrival (AoA) and Time of Arrival (ToA) methods to localize a moving receiver. The assumptions here are synchronized clocks between senders and the receiver and no drastic change of the direction of movement. We consider unsynchronized clocks.

The TDoA localization problem of moving receivers has been analyzed in combination with the frequency differences of arrival (FDoA) technique to estimate the position of a fixed [8] or moving source [9], [10]. An approach in which the sender is not fixed and the receiver location is not exactly known was presented in [11]. Our setting is distinguished from FDoA, as discrete measurements are received, so no continuous comparison of the phase is possible.

We focus on systems based on TDoA. In popular approaches the position of the receiver is tracked. The algorithms used are squared or maximum likelihood estimators [12], particle filters [2], [13] or Kalman filters [14], [15]. We apply the particle filter and the unscented Kalman filter, which has a slightly better performance than the extended Kalman filter in non-linear settings [16].

III. Problem Formulation

We consider the localization problem of a receiver moving on an unknown trajectory in two-dimensional Euclidean space. $\mathbf{M}_t \in \mathbb{R}^2$ denotes the position of the receiver at time t.

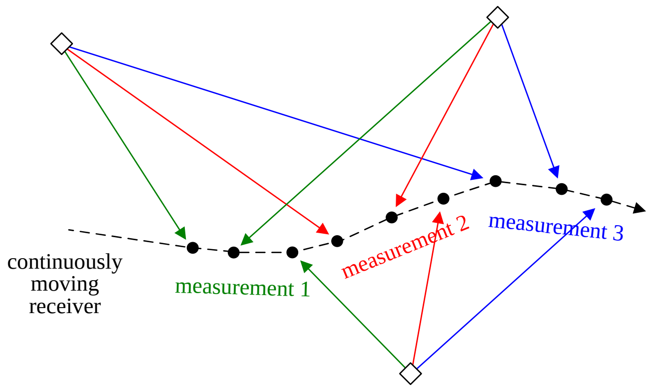

Fig. 1. Schematic of the under-determined equation system: In the example a receiver moves continuously, obtaining three signals from three senders (white diamonds), yielding nine measurements. The measurements are received at nine unknown receiver positions (black dots), which results in 18 unknown variables only for the receiver.

The scenario also consists of m stationary senders which are placed randomly at unknown positions $\mathbf{S}_i$ $(1 \le i \le m)$. Every sender emits discrete signals at regular points in time at a fixed interval I_i. The interval may differ from sender to sender. The sending time of the k_i-th signal at sender position $\mathbf{S}_i$ is then described by

$$t_{k_i i} = t_{0i} + k_i I_i \ , \qquad (k_i > 0). \tag{1}$$

Furthermore, we assume that a k_i-th signal of sender $\mathbf{S}_i$ propagates in a straight line from the sender to the receiver and is received at time point

$$T_{k_i i} = \frac{1}{c}\|\mathbf{M} - \mathbf{S}_i\| + t_{k_i i}, \tag{2}$$

where c is the signal velocity and $\|\cdot\|$ denotes the Euclidean norm. Furthermore, we assume that the signals can be distinguished, as explained previously.

The computation of the length of the intervals of the different senders by the receiver is straightforward by receiving two or more successive signals $k_1, k_2, k_1 \neq k_2$, emitted by the same sender $\mathbf{S}_i$ while it is temporarily stationary. Then the interval for sender $\mathbf{S}_i$ is

$$I_i = \frac{1}{k_1 - k_2}(T_{k_1 i} - T_{k_2 i}) \ . \tag{3}$$

The sending timestamps, as well as the positions of the senders and the current receiver position, remain unknown up to now, and only the signal speed c and the time when a signal has been received, T, are known. To successfully compute the position of the receiver more effort is needed.

In addition to this, the senders are assumed to be unsynchronized, i.e. they send the signals after different intervals and have a different initial send time t_{0i}. Consequently, there is an unknown time offset δ_{ij} between the senders, which describes the send time difference between senders $\mathbf{S}_i$ and $\mathbf{S}_j$. It is calculated by

$$\delta_{ij} = t_{0i} - t_{0j} = (t_{k_i i} - k_i I_i) - (t_{k_j j} - k_j I_j) \ . \tag{4}$$

If all $\binom{m}{2}$ offsets are known or equal to zero, the senders are considered synchronized. Since the offsets are transitive, only $m-1$ offsets must be computed, relative to one sender. Considering the case that the receiver is continuously moving, signals are received at different positions. This results in the following hyperbolic equation in which two signals, originating from two different senders $\mathbf{S}_i$ and $\mathbf{S}_j$, are received at the positions $\mathbf{M}_{k_i i}$ and $\mathbf{M}_{k_j j}$:

$$\frac{1}{c}(\|\mathbf{M}_{k_i i} - \mathbf{S}_i\| - \|\mathbf{M}_{k_j j} - \mathbf{S}_j\|) = \Delta t_{ij} + \delta_{ij} \ , \qquad (5)$$

where Δt_{ij} represents the unsynchronized time difference of arrival of the two signals originated by $\mathbf{S}_i$ and $\mathbf{S}_j$, which may be calculated based on the reception times and the intervals as

$$\Delta t_{ij} = (T_{k_i i} - T_{k_j j}) - (k_i I_i - k_j I_j) \, . \qquad (6)$$

In this equation is m the number of senders, so $m-1$ offsets are required. Assuming the intervals I_i and I_j are known, there exist $2mn + 2m + (m-1)$ unknown variables after n received signals from each sender, but only mn time measurements (cf. Fig. 1). One can see that this equation system is under-determined and cannot be solved in closed form without further information or assumptions on the scenario.

IV. Sender localization

Since the sender positions are assumed to be unknown in the beginning, they need to be calculated. The calculation of senders is based solely on evaluation of the observed time differences by the receivers. This eliminates the need to measure the senders by hand, enabling the user of the localization system to install them at arbitrary places. In this way a "plug-and-play" localization system is created. Calculation of the sender positions is a onetime task. Once the sender positions are known, the position information is downloaded by the receivers, so they may localize themselves with respect to the senders.

For localization of the senders only by measurements of unknown receivers, an adaption of two approaches may be applied, the Ellipsoid TDoA method [17] and statistical approaches that estimate the distance between senders based on the distribution and minima/maxima of the observed reception times [18], [19], [20]. For the statistical algorithms the receiver has to be brought close to every sender, such that the receiver and every pair of senders is aligned and the maximum possible time difference is produced. Alternatively, the receiver moves on a trajectory beyond the vicinity of the senders. If the receiver moves slowly in comparison to the signal velocity, an approximation of the time differences at certain points can be measured.

To compute the distances between sender pairs the Ellipsoid TDoA method uses linear regression to calculate an approximation of the distances of three senders $\mathbf{S}_i$, $\mathbf{S}_j$ and $\mathbf{S}_k$ based on the ellipsoid equation

$$ax^2 + by^2 + cxy + dx + ey = 1 \ , \qquad (7)$$

given the observed TDoA measures. This equation can be transformed into a translation-invariant form

$$\hat{a}(x - \hat{d})^2 + \hat{b}(y - \hat{e})^2 + \hat{c}(x - \hat{d})(y - \hat{e}) = 1 \ , \qquad (8)$$

where $\hat{a}, \hat{b}, \hat{c}$ are translation-invariant parameters that characterize the ellipse, and $(\hat{d}, \hat{e})$ is the shift of the ellipse center, which equals the synchronization offset of the senders. One can calculate an approximation of the distances $d_1 \approx \|\mathbf{S}_i - \mathbf{S}_j\|$ and $d_2 \approx \|\mathbf{S}_i - \mathbf{S}_k\|$ based on the parameters $\hat{a}, \hat{b}, \hat{c}$ from Eq. (8) by

$$d_1 = 2\sqrt{\frac{\hat{b}}{4\hat{a}\hat{b} - \hat{c}^2}} \qquad d_2 = 2\sqrt{\frac{\hat{a}}{4\hat{a}\hat{b} - \hat{c}^2}} \ . \qquad (9)$$

Using the "MinMax" procedure, therefore evaluating the minimum and maximum occurring TDoA, the time difference of every measurement point and the first point is calculated by

$$\tau_{si} = T_{si} - T_{0i} - kI_i \ . \qquad (10)$$

T_{si} describes the timestamp of the signal which has been received at measurement point s, and T_{0i} describes the timestamp for the first point, respectively. Once the time shift has been determined for all stops and all senders the difference of these shifts is be calculated for each stop by

$$\Delta t_{sij} = \tau_{si} - \tau_{sj} \ . \qquad (11)$$

This calculation results in sets of time differences for each sender pair, $\mathcal{K}_{ij} = \{\Delta t_{0ij}, ..., \Delta t_{sij}\}$. The distance between two of those senders is an approximation of the maximum and minimum values of the respective set for senders $\mathbf{S}_i$ and $\mathbf{S}_j$:

$$\|\mathbf{S}_i - \mathbf{S}_j\| \approx d_{ij} = \frac{c}{2}\Big(\max(\mathcal{K}_{ij}) - \min(\mathcal{K}_{ij})\Big) \qquad (12)$$

After one of the distance approximation procedures has been applied, where the choice of the algorithm depends on the flexibility of receiver movements, and the availability of calibration measurements, the distances are known and the senders can be placed in a coordinate system by solving a problem of multi-dimensional scaling [21], [22].

The precision of the Ellipsoid TDoA method depends on the *far-field assumption* of the receiver, therefore if the receiver is too close beneath the senders, an error is induced into the distance calculation, leading to an under-estimation of the sender distances. The precision of the MinMax procedure depends on the availability of measurements on the "long edges" of the senders, therefore an alignment of each pair of two senders and the receiver on a line. As only a single maximum time difference is evaluated, the MinMax procedure is also prone to measurement errors. Such errors may be compensated by error mitigation algorithms such as RANSAC or by estimation of a trustable bound as in PANDAA [23].

V. PROBABILISTIC STATE ESTIMATION

As discussed in the problem formulation, the equation system in (5) is under-determined and cannot be calculated directly, as only one equation is available for every receiver position, cf. Fig. 1. Instead, we consider the positioning problem of a continuously moving receiver as a recursive state estimation problem. The Bayesian filtering scheme [24] has been successfully applied to TDoA problems previously, yet for estimation of a moving sender [2].

The Bayesian filtering scheme is a probabilistic approach to recursive state estimation based on the Markov assumption, i.e. the assumption that the current state depends only on the previous state, not on the previous trajectory. We present an approach based on a stochastic Monte-Carlo simulation, also known as *particle filter*, which is robust to motion and measurement uncertainties, and therefore well suited for TDoA localization. Furthermore, we present an approach based on the unscented Kalman filter, which is an efficient algorithm that is robust even in a non-linear problem setting.

In a recursive Bayesian filter the probability $p(\mathbf{x}_t \mid \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ of state $\mathbf{x}_t$ at time t is assumed to depend on the obtained sensor data $\mathbf{z}_{1:t}$ and control commands $\mathbf{u}_{1:t}$. The posterior distribution, or *belief*, can be described by the recursive update equation

$$\begin{aligned} p(\mathbf{x}_t \mid \mathbf{z}_{1:t}, \mathbf{u}_{1:t}) \;&=\; \eta_t\, p(\mathbf{z}_t \mid \mathbf{x}_t) \\ &\int p(\mathbf{x}_t \mid \mathbf{u}_t, \mathbf{x}_{t-1})\, p(\mathbf{x}_{t-1} \mid \mathbf{u}_{1:t-1}, \mathbf{z}_{1:t-1})\, d\mathbf{x}_{t-1} \;, \end{aligned} \tag{13}$$

where the parameter η_t is a normalizing constant ensuring that $\int p(\mathbf{x}_t \mid \mathbf{u}_{1:t}, \mathbf{x}_{1:t})\, d\mathbf{x}_t = 1$. The state transition probability $p(\mathbf{x}_t \mid \mathbf{u}_t, \mathbf{x}_{t-1})$ and the measurement probability $p(\mathbf{z}_t \mid \mathbf{x}_t)$ are specified by the motion model and sensor model, respectively. In our approach, we use the particle filter and the unscented Kalman filter, which are implementations of the Bayesian filtering model. The particle filter uses a set of *particles* to represent a state hypothesis, approximating the current belief. The unscented Kalman filter uses the knowledge of the noise factors involved in the system to estimate a Gaussian probability function of the state. The non-linear functions, which are characteristical for TDoA, are linearized in the *unscented transform*.

For both algorithms we estimate the same state, consisting of the position vector $\mathbf{M}_t$ of the receiver at time t, as well as the movement velocity of the receiver $\mathbf{V}_t$.

Since the senders are unsynchronized, the time offsets δ_{ij} need to be estimated as well. In order to calculate the offset between $\mathbf{S}_i$ and $\mathbf{S}_j$ with respect to the sending times t_{0i}, t_{0j} we use the equation

$$\delta_{ij} = (k_i I_i + t_{0i}) - (k_j I_j + t_{0j}) + \mathcal{U}(-t_{\text{dist}}, t_{\text{dist}}) \;, \tag{14}$$

where $\mathcal{U}$ is uniformly distributed noise in a time range of t_{dist}. As the offsets are transitive, we settle to estimate $m-1$ offsets. Without loss of generality we estimate only the offset relative to sender $j = 1$ and define $\delta_i = \delta_{1i}$, where $\delta_1 = 0$.

Also the sending time of one sender must be estimated, as in TDoA only relative distances are measured. To propagate the estimated sending time of the latest received signal of sender i we use the last estimated sending time, the interval length and a uniform distributed noise term as

$$t_{k_i i} = t_{(k_i-1)i} + I_i + \mathcal{U}(-t_{\text{dist}}, t_{\text{dist}}) \;. \tag{15}$$

We add the uniformly distributed noise $\mathcal{U}$ to increase the variance of the estimated offsets. As the send times $t_{k_i i}$ are all relative we estimate only $t_{k_1 1}$. Altogether, the state vector is

$$\mathbf{x}_t = \left(\mathbf{M}_t^T, \mathbf{V}_t^T, t_{k_1 1}, \delta_1, ..., \delta_m\right)^T \;. \tag{16}$$

The algorithms use a motion model to propagate the state hypothesis in the current time step based on the previous belief, as well as a sensor model which determines the likelihood of observed measurements. The design of the motion model is crucial for the proper estimation of the state transition, and therefore for efficiency and accuracy of the localization. For both algorithms we use the same motion model, which is based on the model in [2]. In the model, we assume that no control over the movement of the receiver is given, hence the control command represents just the time which has passed since the last computation. For the movement, and therefore the next estimated position of the receiver, we use a constant velocity model. This model assumes that the receiver moves with constant velocity, while changes in the velocity are undetermined, which is modeled by Gaussian noise with a covariance matrix Σ_V. In this model, the position and velocity of the receiver are updated according to the Euler integration scheme by

$$\begin{aligned} \mathbf{M}_{t+1} &= \mathbf{M}_t + h_t \mathbf{V}_t \\ \mathbf{V}_{t+1} &= \mathbf{V}_t + \zeta_t \;, \qquad\qquad \zeta_t \sim \mathcal{N}(0, \Sigma_V) \end{aligned} \tag{17}$$

where $h_t = T_{k_i i} - T_{k_j j} > 0$ is the time between the current signal from sender i and the previous signal from sender j.

A. Particle filtering

The particle filter, also known as Monte Carlo localization [25], recursively approximates the current system state based on the previous system state. For an explanation of the particle filter, see also [2]. A set of particles of size N represents the current belief $p(\mathbf{x}_t \mid \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$. Each particle $(\mathbf{x}_t^{[h]}, w_t^{[h]}), (1 \leq h \leq N)$ represents a hypothesis of the system state at time t and consists of a system state $\mathbf{x}_t^{[h]}$ and the so-called importance weight $w_t^{[h]}$. Using the motion model, the particle filter estimates the values in the current step. The sensor model computes the probability $p(\mathbf{z}_t \mid \mathbf{x}_t^{[h]}, w_{t-1}^{[h]})$ of the measured information $\mathbf{z}_t$ given the state $\mathbf{x}_t^{[h]}$ and the previous importance weight $w_{t-1}^{[h]}$.

The recursive belief update of the particle filter is done according to the following three steps:

1. In the *prediction step* the hypothetical state of a particle $\mathbf{x}_t^{[h]}$ at time t is estimated by drawing a successor state based on the proposal distribution $p(\mathbf{x}_t^{[h]} \mid \mathbf{u}_t, \mathbf{x}_t^{[h]})$

specified by the motion model. The motion uncertainty and the control command $\mathbf{u}_t$ are taken into account in the *prediction step*.

2. In the *correction step* a current measurement $\mathbf{z}_t$ is used to update the weight, $w_t^{[h]} \propto w_{t-1}^{[h]} p(\mathbf{z}_t \mid \mathbf{x}_t^{[h]})$, of each particle. Therefore the likelihood of the state hypothesis is computed using the sensor model and $\mathbf{z}_t$.
3. In the *resampling step* a set of N particles is drawn, replacing the weighted state hypothesis, i.e. the probability of drawing a particle is proportional to its weight.

In our proposed implementation, *resampling* is executed if the effective number of particles N_{eff}, is smaller than the number of $\frac{N}{2}$ particles, as shown in [26], where

$$N_{\text{eff}} = \left(\sum_{h=1}^{N} \left(w_t^{[h]} \right)^2 \right)^{-1} . \quad (18)$$

The particles represent the estimated state as described in Eq. (16). The sensor model uses the measurement $z_j = T_{k_j j}$, which is the timestamp of last received signal originating from sender j, to compute the probability that the observed measurement matches the current belief. Here, we assume that the different measurements are independent, given the current state $\mathbf{x}_t$ of the system. Therefore, the probability of the current measurement, given the system state $\mathbf{x}_t$, is the product of all measurements:

$$p(\mathbf{z}_t \mid \mathbf{x}_t) = \prod_{j=0}^{k} p(z_j \mid \mathbf{x}_t) \quad (19)$$

To evaluate the estimated values, each measurement z_j is taken into account. Based on the known sender positions $\mathbf{S}_j$ and the estimated values $\mathbf{M}_t$, $t_{k_1 1}$, and δ_j, a hypothesis of the observation is calculated by

$$d_{tj} = \frac{1}{c} \|\mathbf{M}_t - \mathbf{S}_j\| + (t_{k_1 1} + \delta_j) . \quad (20)$$

Using this hypothesis d_{tj} the likelihood of a measurement is calculated by

$$p(z_j \mid \mathbf{x}_t) = \mathcal{N}\big(z_j, d_{tj}, \sigma^2_{\text{sensor}}\big) , \quad (21)$$

where σ^2_{sensor} is a variance estimation of the sensor noise, which is assumed to be Gaussian distributed.

B. Unscented Kalman filtering

The unscented Kalman filter (UKF), which was proposed by Julier and Uhlman [27], is a recursive state estimator based on the unscented transform [24], [28], an approach to linearization of non-linear models. For a random variable with dimension L, $2L+1$ “sigma points” are generated deterministically with the known covariance matrix of the involved variables and the tuning parameters of the filter. As the evaluation of the models is calculated only for the $2L+1$ sigma points, the UKF is cheaper in computation than the particle filter, which requires evaluation of a large number of particles, or the extended Kalman filter, which requires calculation of the Jacobian matrix. For our implementation of the UKF for tracking of a moving receiver by unsynchronized TDoA we follow the description given in [24].

1) Generating sigma points: First, an extended mean vector and covariance matrix are generated:

$$\begin{aligned} \mu^a_{t-1} &= \begin{pmatrix} \mu^T_{t-1} & \mu^T_V & \mu^T_m & \mu^T_o \end{pmatrix}^T \\ \Sigma^a_{t-1} &= \begin{bmatrix} \Sigma_{t-1} & 0 & 0 & 0 \\ 0 & \Sigma_V & 0 & 0 \\ 0 & 0 & \sigma^2_m & 0 \\ 0 & 0 & 0 & \Sigma_o \end{bmatrix} \end{aligned} \quad (22)$$

where Σ_o is the covariance matrix of the offset noise, σ^2_m is the variance of the measurement noise and $\mu_{t-1}, \ \mu_V, \ \mu_m, \ \mu_o$ are the mean of the previous state, the velocity noise, the measurement noise and the offset noise, respectively. The velocity and the offset noise are called "process noise".

The augmented mean and variance are used to generate a set of $2L$ sigma points, where L is the dimension of the augmented mean vector:

$$\chi^a_{t-1} = \begin{bmatrix} \mu^{a\,T}_{t-1} \\ \left(\mu^a_{t-1} + \sqrt{(L+\lambda)\Sigma^a_{t-1,[1]}}\right)^T \\ \vdots \\ \left(\mu^a_{t-1} + \sqrt{(L+\lambda)\Sigma^a_{t-1,[L]}}\right)^T \\ \left(\mu^a_{t-1} - \sqrt{(L+\lambda)\Sigma^a_{t-1,[1]}}\right)^T \\ \vdots \\ \left(\mu^a_{t-1} - \sqrt{(L+\lambda)\Sigma^a_{t-1,[L]}}\right)^T \end{bmatrix}^T , \quad (23)$$

where $\Sigma^a_{\bullet,[\ell]}$ denotes the ℓ-th column of $\Sigma^a_{\bullet}$. The scaling parameter

$$\lambda = \alpha^2(L+\rho) - L \quad (24)$$

determines how far the sigma points are from the mean, where α and ρ are tuning parameters of the filter.

2) Prediction: The sigma points matrix has now a dimension of $L \times (2L+1)$. This matrix has rows in the space of the previous state, the measurement noise and the process noise:

$$\chi^a_{t-1} = \begin{bmatrix} \chi^x_{t-1} & \chi^m_t & \chi^o_t & \chi^V_t \end{bmatrix}^T \quad (25)$$

Each row has $2L+1$ sigma points. The sigma points of the previous state and the process noise are passed to a function g, which is the motion model defined in Eqns. (14) and (17), to predict the new state:

$$\chi^x_t = g(u_t, \chi^x_{t-1}, \chi^V_t, \chi^o_t) \quad (26)$$

After that, the Gaussian statistics of the new points are computed by

$$\overline{\mu}_t = \sum_{l=0}^{2L} w_m(l) \chi^x_{l,t} , \qquad \overline{\Sigma}_t = \sum_{l=0}^{2L} w_c(l) \chi^x_{l,t} , \quad (27)$$

where

$$w_m(0) = \frac{\lambda}{L+\lambda}$$
$$w_c(0) = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta)$$
$$w_m(l) = w_c(l) = \frac{1}{2(L+\lambda)} . \qquad (1 \le l \le 2L)$$

The parameter β is a tuning parameter of the filter which has to be set depending on the a priori knowledge of the state probability distribution. The observations at the sigma points are calculated in the sensor model by

$$\overline{z}_t = h(\chi_t^x) + \chi_t^m . \tag{28}$$

In our case, the sensor model follows this equation for every sigma point:

$$\overline{z}_t = \frac{1}{c}\|\mathbf{M}_t - \mathbf{S}_j\| + (t_{k_1 1} + \delta_j) + \chi_t^m \tag{29}$$

3) Correction: In the final step, the Gaussian statistics of the observations at the sigma points and the predicted state are calculated to correct the measurement.

$$\mu_{t,z} = \sum_{l=0}^{2L} w_m(l)\,\overline{z}_{l,t}$$
$$\sigma_z^2 = \sum_{l=0}^{2L} w_c(l)\,(\overline{z}_{l,t} - \mu_{t,z})^2 \tag{30}$$

In this step also the cross-covariance between the predicted state and the predicted measurement is calculated:

$$\Sigma^{x,z} = \sum_{l=0}^{2L} w_c(l)\,(\overline{\chi}_{l,t}^x - \overline{\mu}_t)\,(\overline{z}_{l,t} - \mu_{t,z})^T \tag{31}$$

Finally, the mean and the covariance of the state are updated by

$$\mu_t = \overline{\mu}_t + K_t(z - \mu_z)$$
$$\Sigma_t = \overline{\Sigma}_t - K_t K_t^T \sigma_z^2 , \tag{32}$$

where $K_t = \frac{1}{\sigma_z^2}\,\Sigma^{x,z}$ is the Kalman gain.

VI. Experiments

For evaluation of the algorithms we have implemented both algorithms and analyzed and compared them in extensive numerical simulations. The simulation for the unscented Kalman filter was implemented in Matlab. On grounds of performance, we implemented the particle filter in C++ in order to execute the algorithm with a large number of particles. For both algorithms we use the same set of random test data, the same trajectories of the moving receiver, in an experiment area of 15 m × 15 m. The trajectory was implemented as a polygon of lines with a definite start and end point, where the receiver moves at a velocity of 0.4 m/s. The reception times of signals by the moving receiver were computed in advance, which requires solving an equation system of the reception time and the receiver position at the time of reception. The senders were placed arbitrarily in the area. The sender intervals were chosen arbitrarily between 0.250 s and 0.350 s, with a random offset.

To calculate the mean error and standard deviation of the error of one simulation, for each measurement the distance between the true position and the position estimates is calculated by

$$\mu_\varepsilon = \frac{1}{n}\sum_{t=1}^{n}\|\mathbf{M}_t - \overline{\mathbf{M}}_t\|$$
$$\sigma_\varepsilon^2 = \frac{1}{n-1}\sum_{t=1}^{n}\left(\|\mathbf{M}_t - \overline{\mathbf{M}}_t\| - \mu_\varepsilon\right)^2 , \tag{33}$$

where $\mathbf{M}_t$ represents the true position and $\overline{\mathbf{M}}_t$ the estimated position at the arrival of the t-th signal, and n is the total number of received signals in one simulation.

To simulate an error at the receiver, a Gaussian distributed error variable $\xi \sim \mathcal{N}(0, \sigma_\xi^2)$ is added to each timestamp T_{ki}. In the experiments in [2] an ultrasound system was used, where a standard deviation of approximately 0.1 ms was a typical magnitude of error. We decided to use a slightly larger timing noise of σ_ξ = 0.3 ms in order to consider errors due to adverse environment settings, ensuring the robustness of the presented algorithms.

A. Particle filter

For initialization of the particle filter we choose a uniform distribution, as no information of the receiver position is given. The initial position of the receiver $\mathbf{M}'_0$ of each particle is initialized uniformly distributed in an area of q = 30 m edge length by $\mathbf{M}'_0 = \mathcal{U}(-\frac{q}{2}, \frac{q}{2})$. We assume that we could find a rough initialization for the estimated sender time and offset by the algorithms discussed in Section IV, therefore we use the true value, adding a uniform error of $t_r = \frac{1}{c}$ m ≈ 0.003 s:

$$t'_{01} = t_{01} + \mathcal{U}(-t_r, t_r)$$
$$\delta'_j = \delta_j + \mathcal{U}(-t_r, t_r) . \tag{34}$$

For evaluation of the precision of the particle filter three experiments were executed. In the first experiment we analyzed the required number of particles in relation with the computation time. In the second experiment we added an artificial measurement error to evaluate the robustness and ability of the particle filter to compensate for larger errors. In the third experiment we varied the number of senders and analyzed impact on the quality of localization.

For every experiment the algorithm was run 100 times using a random trajectory, and random sender positions and intervals. For evaluation of the particle filter we consider the median of errors, which is more reliable than the arithmetic mean, which for example suffers from very large errors during the initialization phase. In some rare cases (about 1%) the algorithm failed to initialize correctly. We eliminated these failed attempts by hand.

In the *first experiment* with variation of the number of particles, the particle filter has been executed with four senders and no synthetic TDoA error. In Fig. 2 the median error and the median computation time for different numbers of particles are shown. Computation time describes the time for algorithm cycle, i.e. one execution of the prediction and correction

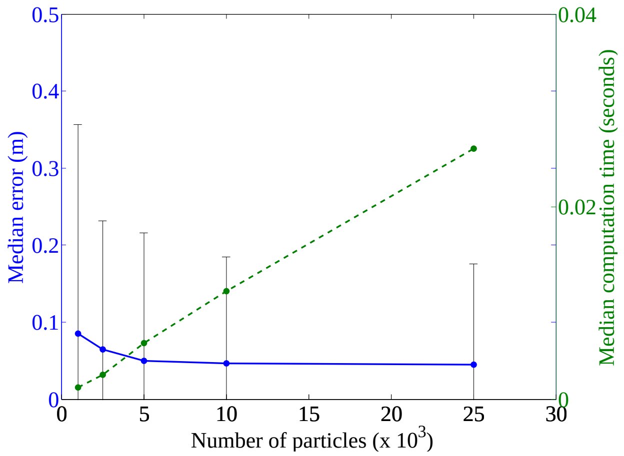

Fig. 2. Median error (solid line) and computation time of one iteration of the particle filter (dotted line) with increasing number of particles. The number of particles is chosen as a tradeoff of localization error and computation time. In our experiments we use 5000 particles.

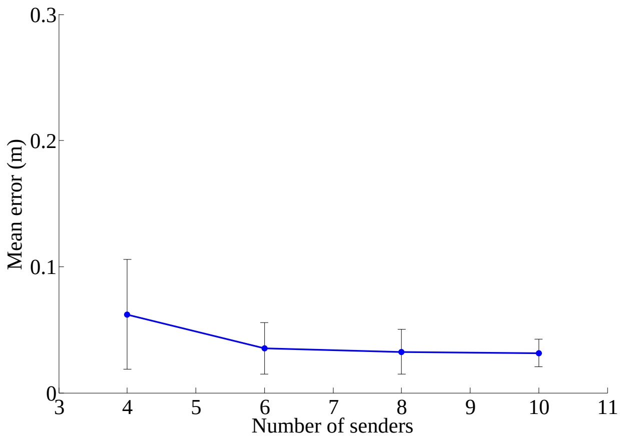

Fig. 4. Mean of median localization errors for each run, using different numbers of senders. Three senders is the required minimum. A larger number of senders compensates for measurement errors. More than six senders do not further decrease the error due to motion uncertainty of the receiver.

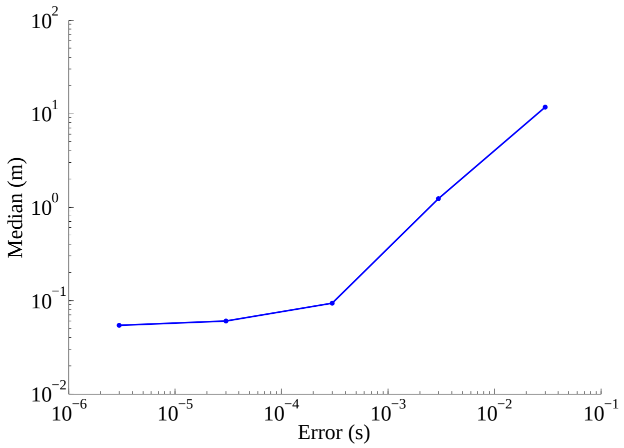

Fig. 3. Median localization error for increasing Gaussian TDoA error of the reception time. The localization error is proportional to the TDoA error, if the TDoA error is larger than 0.001 s. Below this magnitude the intrinsic error of the particle filter dominates the localization error.

steps and a resampling step if necessary. It can clearly be seen that with increasing number of particles the error in the localization decreases to a certain degree, where a larger number of particles does not result in further improvement of precision. As indicated by the median of the computation time, the required computation time is linear in the number of particles. One would choose a particle count where the result is sufficiently good and the computation time is still in a reasonable range. The proper tradeoff is of particular importance when running the algorithm in real-time on embedded hardware where computational power is scarce.

In the *second experiment* Gaussian distributed errors were added to the true reception time. Here, we used 5000 particles, a tradeoff result from the first experiment, and four senders located at benign positions. TDoA error values were increased from $0.3 \cdot 10^{-5}$ to $0.3 \cdot 10^{-1}$ seconds standard deviation. Fig. 3 illustrates the resulting position error in meters for the given measurement errors in the timestamp. The results indicate that the localization error of the receiver is proportional to the measurement error, if above a certain level of approximately 0.001 s standard deviation. Below this threshold for decreasing errors, the localization error converges to the intrinsic localization error of the particle filter, which originates from randomized uncertainty in the receiver motion. According to these results the particle filter is robust even for large measurement errors of 0.01 s, which corresponds to a distance equivalent of 3.5 m, where still plausible tracking reliability is achieved.

In the *third experiment* a varying number of senders was used to evaluate the influence of the number of senders on the localization error. Fig. 4 shows the localization error and the standard deviation for four to ten senders. No artificial measurement error was added in this experiment. Three senders is the theoretical minimum required for the TDoA equation system to determine a unique trajectory in two dimensions. A larger number of senders allows for more reliable weighting in the importance sampling step, therefore compensating and decreasing localization error of the mobile receiver. In our settings, more than six well distributed senders do not further decrease the localization error if no measurement error is given, as uncertainty in the prediction of the receiver motion dominates the localization error. Furthermore, the computation time is increased by processing the signals of many senders, as the interval of subsequent signals from the senders is decreased.

B. Unscented Kalman filtering

Employment of the Kalman filter requires good knowledge of the process noise variance and the measurement noise variance. The Kalman parameters are dependent on the parameters of the simulation, which are the velocity and trajectory of the receiver, the distribution of measurement error, and the senders positions and signal intervals. We set a fixed Gaussian TDoA error of the timestamps of $\sigma_\xi = 0.3$ ms, as noted previously.

TABLE I
MEAN ERROR AND STANDARD DEVIATION FOR VARYING NUMBERS OF SENDERS IN THE UKF EXPERIMENT.

	3 senders	4 senders	8 senders
Mean error (m)	0.115	0.104	0.073
Standard deviation (m)	0.064	0.059	0.042

The senders were located at remote positions of the field (Table III), to reduce the effect of adverse sender positioning.

We ran an experiment of the UKF variating both parameters, while adding the Gaussian error of to the true measurements. The results in Fig. 5 indicate that localization precision decreases slightly if the process noise variance is overestimated, however the UKF becomes unstable if process noise is underestimated, as the receiver estimate cannot follow the curvy trajectory. Now, the experiment was repeated 100 times, fixing the process noise variance to $\sigma_V^2 = 0.120\,\text{m}^2$, for each magnitude of the measurement noise variance. The result is shown in Fig. 6. When the parameters are properly adjusted, the mean error is reduced to 0.115 m with a standard deviation of 0.064 m. The minimum median error is 0.106 m.

The experiments of the unscented Kalman filter were executed assuming that we know the initial position and velocity of the receiver, and the process noise variance is:

$$\Sigma_V = \begin{pmatrix} \sigma_V^2 & 0 \\ 0 & \sigma_V^2 \end{pmatrix} \tag{35}$$

The error can be further reduced either increasing the number of senders to some extent or reducing the interval. We have executed two experiments to evaluate these effects, using the optimum parameters for each scenario, and with the same measurement error as before. In the first experiment we have tested the algorithm for a varying number of senders. We located the senders as described in Table III as far as possible from each other and we have used with sending intervals I_i for sender i randomized from 0.250 s to 0.350 s. These intervals are fixed for every simulation. The results are displayed in Table I. It can be seen that every additional sender causes a slight reduction of the error.

In the second experiment the intervals are multiplied by 0.1 and 0.01. As shown in Table II, the error is notably reduced when the interval is multiplied by 0.1. Yet, multiplying by 0.01 has almost no benefit compared to multiplying with 0.1, only the standard deviation of the error is slightly reduced. The risk of a severely decreased interval size is the increase of computation time, as the Kalman filter iterates for every received signal, and the increasing difficulty to distinguish signals of a certain length in practical application.

C. Comparison

To compare the two algorithms and the precision of their localization, we executed a simulation of the particle filtering and the unscented Kalman filtering approach in identical scenarios. We used a fixed trajectory for both algorithms, traversing the entire experimental field of 15 m × 15 m, with a fixed number of three well-placed receivers. The positions and the intervals of the senders are listed in Table III.

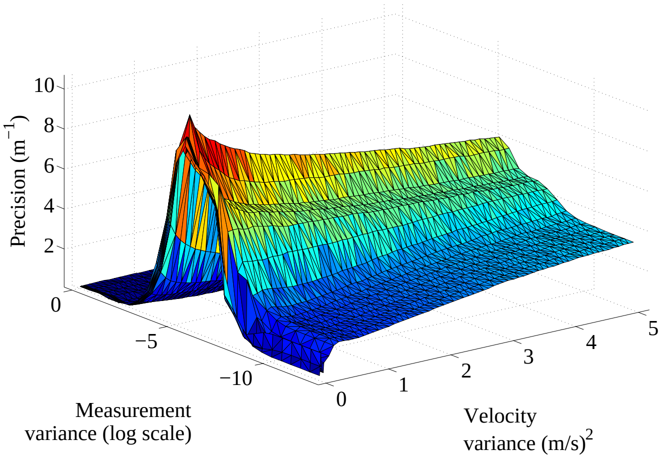

Fig. 5. Inverse of the UKF median error with respect to different parameter settings of process noise variance and measurement noise variance. The measurement variance is represented on a log scale by $\log_{10}(\sigma_m^2)$. Good knowledge of the process and measurement errors increases the precision of receiver localization immensely.

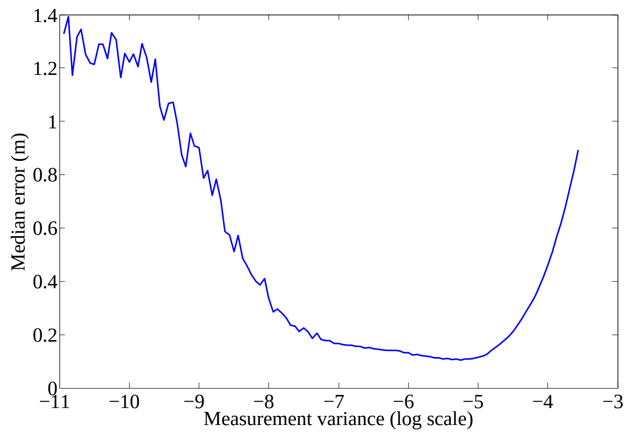

Fig. 6. Fixed process noise variance $\sigma_V^2 = 0.12\,\text{m}^2/\text{s}^2$. The measurement variance is represented on a log scale by $\log_{10}(\sigma_m^2)$. If the measurement variance is fixed between 10^{-6} and 10^{-5} the median localization error is below 0.134 m.

For this comparison both algorithms were given measurements with a Gaussian distributed error with standard deviation of $\sigma_\xi = 0.3\,\text{ms}$, which is added to the reception times. The initialization for the particle filter and the UKF was chosen more precise with an uniform error of 1 m added to the true receiver position. The initialization of the sending time and the offsets were chosen the same as in the previous experiments.

An algorithm run on the static trajectory was run given these settings. The resulting trajectory and estimations are displayed in Fig. 7. According to the results, both algorithms are clearly capable of localizing the receiver, based on the incoming signals, even though these are subject to Gaussian measurement errors, with a moderate advantage of the particle filter. Apparently, both algorithms show a drag towards the outside of a turn, which is a result of the constant velocity

TABLE II
MEAN ERROR AND STANDARD DEVIATION WHEN THE INTERVAL IS REDUCED IN THE UKF EXPERIMENT. DECREASING THE INTERVALS TO 0.1 I_i REDUCES LOCALIZATION ERROR NOTABLY, YET AT A HIGHER COMPUTATIONAL COST. REDUCING THE INTERVAL EVEN FURTHER YIELDS ALMOST NO BENEFIT.

	I_i	$0.1I_i$	$0.01I_i$
Mean error (m)	0.115	0.069	0.069
Standard deviation (m)	0.064	0.048	0.042

TABLE III
SENDER POSITIONS AND INTERVAL LENGTHS FOR THE SENDERS USED IN THE COMPARATIVE RUN OF THE PARTICLE FILTER AND THE UNSCENTED KALMAN FILTER.

	$\mathbf{S}_1$	$\mathbf{S}_2$	$\mathbf{S}_3$
Position x (m)	4	15	0
Position y (m)	0	11	15
Interval (s)	0.255	0.300	0.350

motion model, assuming a straight path. Therefore the estimation exhibits an inertia until the measurements suggest the changing direction.

Fig. 8 shows the different error sizes of the particle filter and the unscented Kalman filter according to the received signals. It shows that the estimation of the particle filter has a moderately smaller error with a mean of 0.084 m and a standard deviation of 0.046 m. The estimation of the unscented Kalman filter has a mean error of 0.117 m and a standard deviation of 0.063 m. Since Fig. 9 represents the error of the trajectory in Fig. 7 one can see that the unscented Kalman filter estimate has a weaker precision of the receiver position. This reflects the results of Fig. 8 but also reveals that the error of the UKF is not so much larger than the error of the particle filter, yet at a much lower computational cost.

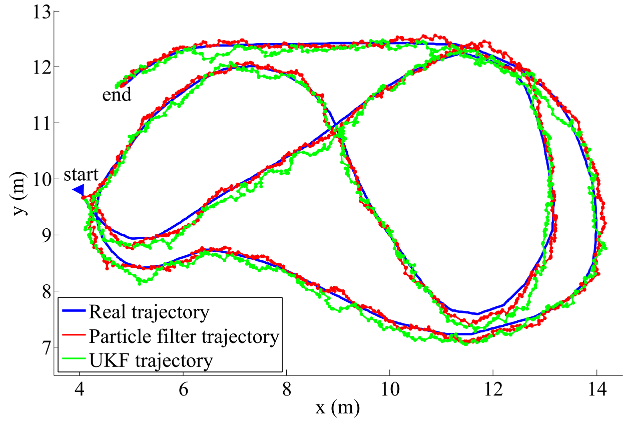

Fig. 7. Comparison of the particle filter (red) and the unscented Kalman filter (green) in a realistic scenario. Both algorithms are well capable to follow the true trajectory of the receiver (blue), with a moderate advantage for the particle filter. A drag towards the outside of a curve is caused by inertia of the constant velocity model.

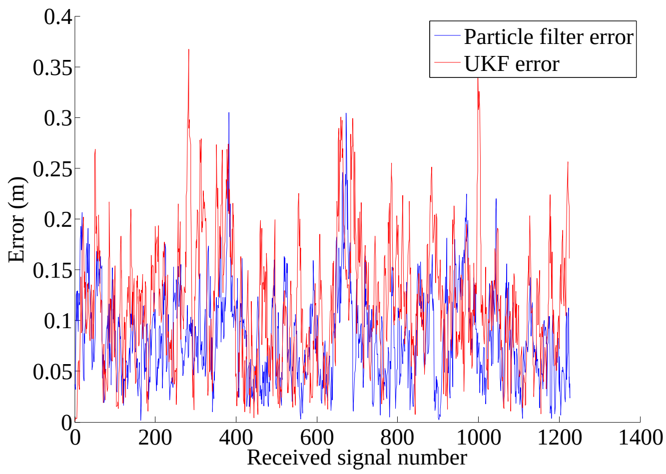

Fig. 8. The localization errors of the trajectory in Fig. 7 for each received signal displayed for the particle filter (blue) and the unscented Kalman filter (red) as a function of time.

VII. CONCLUSIONS

In this paper, we have presented two novel approaches for the localization of a mobile receiver, using the time differences of arrival of unsynchronized senders, and where the sender positions are unknown at the beginning. To address the problem of the sender localization we propose using adaptions of two algorithms, the Ellipsoid TDoA method and the MinMax procedure, for which the receiver has to be brought close to all senders, or move on an arbitrary path around all senders.

To address the localization problem of the mobile receiver, which results in an under-determined state estimation problem with ambiguities in the measurements, we have developed a probabilistic formulation in a particle filter and the unscented Kalman filter. We presented a motion model for both algorithms, assuming continuous movement of the receiver, and a sensor model which estimates the sending timestamp of one sender and the time offsets of the senders, effectively solving the synchronization problem.

In the experiments, which have been run for the two probabilistic algorithms, we have shown that the particle filter is capable of precise localization and tracking of the receiver position, while simultaneously estimating the synchronization offsets of the senders. While the particle filter shows minor localization error, it requires high computational effort due to the large number of samples [29]. We also demonstrated that the under-determined problem can be addressed by the unscented Kalman filter with only moderate increase in localization error.

The fact that in our experiments the error of the UKF is higher than with the particle filter differs from the results in [29], where the "standard TDoA" case of tracking a moving *sender* is considered, and where a smaller error is achieved by using UKF, compared to the particle filter. However, in their setting the position of a new signal is uniquely determined by multiple constraints, if a sufficient number of measurements are received. This makes the problem easier than ours, where only one constraint is available at any time. For our scenario, choosing one algorithm or the other would

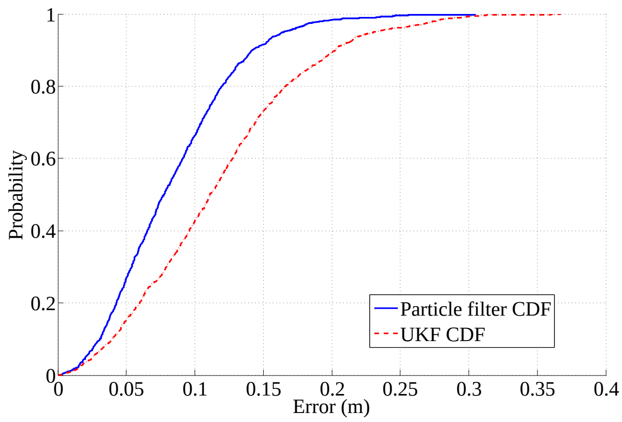

Fig. 9. The cumulative distribution function of the receiver error with the particle filter (solid line) and the unscented Kalman filter (dotted line). The particle filter has a moderate advantage over the unscented Kalman filter.

depend on the requirements of the scenario and the available computational time.

For our future work, we plan to apply an optimization based initialization algorithm for the particle filter and the unscented Kalman filter. Furthermore, we plan to consider experiments in three dimensions, for which the implementation is straightforward using our proposed algorithms, as all components require just increase of the state vector size. We also consider simultaneous localization of the receiver trajectory, the sender offsets, and of the unknown sender positions in an integrated approach using the particle filter, similar as demonstrated in [2] for the standard TDoA setting.

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