Brief Announcement: Network Synchronization and Localization Based on Stolen Signals

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ABSTRACT

We consider an anchor-free, relative localization and synchronization problem where a set of n receiver nodes and m wireless signal sources are independently, uniformly, and randomly distributed in a disk in the plane. The signals can be distinguished and their capture times can be measured. At the beginning neither the positions of the signal sources and receivers are known nor the sending moments of the signals. Now each receiver captures each signal after its constant speed journey over the unknown distance between signal source and receiver position. Given these nm capture times the task is to compute the relative distances between all synchronized receivers. In a more generalized setting the receiver nodes have no synchronized clocks and need to be synchronized from the capture times of the stolen signals.

Categories and Subject Descriptors: G.1.2 Approximation: Nonlinear approximation

General Terms: Algorithms, Measurement

Keywords: TDOA, localization, synchronization

1. INTRODUCTION

Localization and synchronization are fundamental and well researched problems. In this paper we take a fresh look at this problem. We use the time difference of arrival of abundantly available, distinguishable signal sources of unknown location and timing, which can be received at a set of receivers. Assuming that the senders and receivers are on the plane the task is to find the locations of all receivers.

We assume a uniform distribution in a disk of the same or larger size. After collecting all the time information from all receivers we want to compute the time offsets and positions of all nodes without knowing where or when the random signals are produced. We only assume that we can distinguish the signals and that they reach all nodes of our network.

The survey in [1] describes a selection of range-based approaches. Also of interest is the analysis of the uniqueness of ranged networks [2]. The term *time differences of arrival* (TDOA) describes the reception of an unknown signal without any given range information. Close to our problem is the setting in [3] where only TDOA information is used to locate wireless sensor nodes. An elegant solution for a fixed number of 10 microphones is shown in [4]. However, it does not scale for large numbers of microphones.

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In [5] we present a technique for robust distance estimation between microphones by evaluating the timing information of sharp sound signals. We assume synchronized receivers and signals originating from a far distance, but we have no further information about their location.

2. ESTIMATING DISTANCES

Our distance estimation approach begins with only two receivers. As we have pointed out in [5] it is possible to estimate the distance between two receiver nodes if the signals are uniformly distributed on a circle around the receivers at a large distance. Here, we show that this method also results in a reasonable estimation if the signals are distributed in the same disk where the receivers lie.

Max-Min-Technique.

Given two vertices $i, j\,(1 \le i < j \le n)$ and the relative time differences of the stolen signals: $t_{r_i,s_k} - t_{r_j,s_k}$ for all stolen signals $s_1, \ldots, s_m$, we compute the *estimated distance* $d_{i,j}$ *between* i *and* j as

- $d_{i,j} := \max_k\{|t_{r_i,s_k} - t_{r_j,s_k}|\}$ if the receivers are synchronized and as
- $d_{i,j} := \frac{1}{2}\left(\max_k\{t_{r_i,s_k} - t_{r_j,s_k}\} - \min_k\{t_{r_i,s_k} - t_{r_j,s_k}\}\right)$ if the receivers are not synchronized. The estimated relative time offset will be computed using the time signal $k^* := \arg\max_k\{t_{r_i,s_k} - t_{r_j,s_k}\}$. Then, $t_{r_i,s_{k*}} - t_{r_j,s_{k*}} - d_{i,j}$ yields the approximation of the correction for the clocks at i and j.

First, note that in both cases the estimation is always upper-bounded by the real distance: $\|r_i - r_j\| \ge d_{i,j}$. We now describe a sufficient condition for the accuracy of the estimator. For this we define the ϵ-critical area.

Definition 1 *The ϵ-critical area of two nodes (u, v) is the set of points p in the plane where*

$$\|u - v\| - (\|p - v\| - \|p - u\|) \ \le \epsilon \ .$$

This convex area is bounded by a hyperbola containing the point u. If in this critical area signals are produced, then the distance estimation is accurate up to an absolute error of ϵ.

Lemma 1 *If in both of the ϵ-critical areas of (u, v) and (v, u) signals are produced, then the Max-Min distance estimation $d_{u,v}$ is in the interval $d_{u,v} \in [\|u-v\| - 2\epsilon, \|u-v\|]$.*

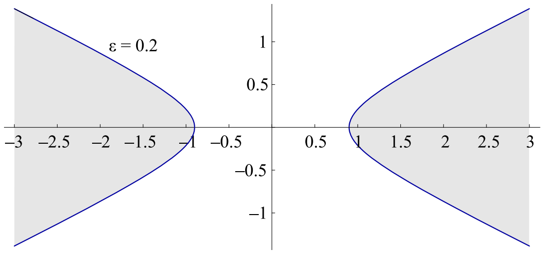

Figure 1: The 0.2-critical areas of two nodes at $(-1, 0)$ and $(1, 0)$ are on the left and right side of the hyperbolas.

The time offset between the clocks of u and v can be computed up to an absolute error margin of 2ϵ.

If at least in one of the ϵ-critical areas of (u, v) and (v, u) a signal is produced, then for synchronized receivers the Max-Min distance estimation $d_{u,v}$ is in the interval $d_{u,v} \in [\|u - v\| - \epsilon, \|u - v\|]$. These signals can be found in time $\mathcal{O}(m)$.

Proof. The proof of the accuracy of the distance estimators follows from the definition of the critical areas. For the accuracy of the time offset consider that one clock u is assumed to be correct, then the other node's clock offset is chosen such that the signal arrives later at time $d_{u,v}$ if the signal was detected at the ϵ-critical area of u.

The best signals can be found by computing the minimum or maximum of the differences of the time points at the receivers u and v. □

Lemma 2 *For two receivers u, v with $\ell := \|u - v\|$ the intersection of the ϵ-critical area (v, u) of a disk with center $\frac{1}{2}(u + v)$ and radius r has*

- *at least an area of $\min\{\pi\ell^2, \frac{1}{2}\epsilon^2\}$ if $r = \ell$ and*
- *at least an area of $\min\{\pi r^2, (r - \ell)^2\sqrt{\epsilon/\ell}\}$ if $r > \ell$.*

Since the critical areas are rather large there is a good chance that a signal could be found in one of these areas.

Theorem 1 *For m stolen signals the Max-Min distance estimator for two receiver nodes u, v with distance $\ell := \|u - v\|$ within the disk with center $(0, 0)$ and radius 1 outputs a result $d_{u,v}$ with $d_{u,v} \in [\|u - v\| - \epsilon, \|u - v\|]$ with probability $1 - p$, where for ϵ and p we have:*

1. *If u and v are unsynchronized and the m signal sources are uniformly distributed in the unit disk we have $\epsilon = \mathcal{O}\left(\sqrt{\frac{\log m}{m}}\right)$ and $p = \frac{1}{m^c}$ for any $c > 1$.*
2. *If u and v are unsynchronized, u and v are not close to the unit disk boundary, i.e. $|u| < 1 - k$ and $|v| < 1 - k$ for some constant $k > 0$, and the m signal sources are uniformly distributed in the unit disk we have $\epsilon = \mathcal{O}\left(\frac{\log^2 m}{m^2}\right)$ and $p = \frac{1}{m^{ck^2}}$ for any $c > 1$.*

Theorem 2 *For unsynchronized receivers we can compute in time $\mathcal{O}(nm)$ an approximation of the positions and the clock offset within an absolute error of $\mathcal{O}\left(\sqrt{\frac{\log m}{m}}\right)$ with probability $1 - m^{-c} - e^{-c'n}$ (for any $c \in \mathcal{O}(1)$ and some $c' > 0$).*

Theorem 3 *For synchronized receivers we can compute in time $\mathcal{O}(nm)$ an approximation of the correct relative positions within an absolute error margin of $\mathcal{O}\left(\frac{\log^2 m}{m^2}\right)$ with probability $1 - m^{-c} - e^{-c'n}$. This error bound holds also for unsynchronized receivers if we consider a normal distribution of the sound signals, or if the sound signals are randomly distributed in a surrounding larger disk.*

In an ad hoc network a distributed algorithm can approximate the positions and clock offsets for the network within an absolute error of $\mathcal{O}\left(\sqrt{\frac{\log m}{m}}\right)$ with probability $1 - n^{-c}$ if $m > n$ using $\mathcal{O}(nm \log n)$ messages.

3. OUTLOOK

From now on, we will use the distance estimation information and compute the locations of the receiver nodes. While our focus is the localization and synchronization of the receivers, it is straight-forward to determine the time and position of the signals using the receivers as anchor points.

Anchor-free localization based on TDOA signals faces the characteristic problems of possible ambiguous solutions and an incoherent solution set. Combined with the non-linear, non-convex nature of this optimization problem (which can be expressed as a set of nm polynomial equations of quadratic degree) one may expect an ill-posed problem. In this paper we overcome this problem with an efficient approximation algorithm. Moreover, we can prove the quality of the result for a random input set with high probability.

The output of our algorithm can be used as the initial starting point of standard non-linear optimization methods like gradient-based search or Newton's method. Our algorithm computes the start position in time $\mathcal{O}(mn)$, i.e. in linear time with respect to the problem size. So, computing such a starting point amounts for the same time as a constant number of iterations for these problems. Whether the success rate of optimization algorithms improves with such an input set is part of further research.

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